ZØ: An Optimizing Distributing Zero-Knowledge Compiler

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Abstract

Applications increasingly rely on privacy-sensitive user data, but storing user's data in the cloud creates challenges for the application provider, as concerns arise relating to the possibility of data leaks, responding to regulatory pressure, and initiatives such as DoNotTrack. However, storing data in the cloud is not the only option: a recent trend explored in several recent research projects has been to move functionality to the *client*. Because execution happens on the client, such as a mobile device or even in the browser, this alone provides a degree of privacy in the computation, with only relevant data disclosed to the server. However, in many cases moving functionality to the client conflicts with a need for computational *integrity*: a malicious client can simply forge the results of a computation.

Traditionally, confidentiality and integrity have been two desirable design goals that are have been difficult to combine. *Zero-Knowledge Proofs of Knowledge* (ZKPK) offer a rigorous set of cryptographic mechanisms to balance these concerns. However, published uses of ZKPK have been difficult for regular developers to integrate into their code and, on top of that, have not been demonstrated to scale as required by most realistic applications.

This paper presents ZØ (pronounced "zee-not"), a compiler that consumes applications written in C# into code that automatically produces scalable zero-knowledge proofs of knowledge, while automatically splitting them into distributed code. ZØ builds detailed cost models and uses two existing zero-knowledge back-ends with varying performance characteristics to select the most efficient translation. Using a set of realistic applications that perform tasks such as distributed data mining and crowd-sourced aggregation, we demonstrate ZØ's ability to produce code that executes significantly faster than was previously possible. Our case studies have been directly inspired by existing sophisticated widely-deployed commercial products that require both privacy and integrity. The performance delivered by ZØ is as much as 58× faster (about 15× on average) than either of the underlying techniques used in the back-ends can deliver, showing that applications in need of ZKPK which were previously hopelessly slow are now within reach for practical deployment.

1 Introduction

As popular applications rely on personal, privacy-sensitive information about users, factors such as legal regulations, industry self-regulation, and a growing body of privacy-conscious users all pressure developers to respond to demands for privacy. Storing user's data in the cloud creates downsides for the application provider, both immediately and down the road. While policy measures such as DoNotTrack and anonymous advertising identifier become increasingly popular, a recent trend explored in several research projects has been to move functionality to the *client* [15, 20, 41, 44]. Because execution happens on the client, such as a mobile device or even in the browser, this alone provides a degree of privacy in the computation: only relevant data, if any, is disclosed (to a server). However, in many cases, moving functionality to the client conflicts with a need for computational *integrity*: a malicious client can simply forge the results of a computation.

Traditionally, confidentiality and integrity have been two desirable design goals that are have been difficult to combine. *Zero-Knowledge Proofs of Knowledge* (ZKPK) offer a rigorous set of cryptographic mechanisms to balance these concerns, and recent theoretical developments suggest that they might translate well into practice. In the last several years, zero-knowledge approaches have received a fair bit of attention [27]. The premise of zero-knowledge computation is its promise of both privacy *and* integrity through the mechanism cryptographic proofs. However, published uses of ZKPK [4, 6, 8, 9, 22, 40] have been difficult for regular developers to integrate into their code and, on top of that, have not been demonstrated to scale, as required by most realistic applications.

Zero-knowledge example: pay as you drive insurance: A frequently mentioned application and a good example of where zero-knowledge techniques excel is the practice of *mileage metering* to bill for car insurance: pay as you drive auto insurance is an emerging scheme that involves paying a rate proportional to the number of miles driven, either linearly, or using several billing brackets (`MileMeter.com`) [5, 42, 45]. Of course, given that the insurance company knows much about the customer, including their address, if daily mileage data is provided, much can be inferred about user's daily activities, where they shop, etc. [18, 34, 35]. The user in this scheme performs a calculation on their own data, but of course the insurance company wants to prevent cheating. Zero-knowledge proofs provide a way to ensure both privacy and integrity, which involves performing the billing computation on the user's hardware (on the *client*), perhaps, monthly, and providing the insurance company with 1) the final bill and 2) a proof of correctness of the accounting calculation, which can be verified by the insurance company (on the *server*) [4, 21, 39, 43].

What we did: In this paper, we present ZØ, a compiler that consumes applications written in a subset of C# into code that automatically produces scalable zero-knowledge proofs of knowledge, while automatically splitting them into distributed code, to be executed on two (or more) execution tiers. We are building on very recent developments in zero-knowledge cryptographic techniques [19, 36], exposing to the developer the ability to take advantage of these advances. ZØ builds detailed cost models of the code regions that require ZKPK, and uses two existing leading-edge zero-knowledge back-ends with varying performance characteristics to select the most efficient translation, by formulating and solving constrained numeric optimization problems. Our cost modeling takes advantage of the strengths of both back-ends, while avoiding their weaknesses, both for local and global (distributed) optimization. Using a set of realistic applications that perform tasks such as distributed data mining and crowd-sourced aggregation, we demonstrate ZØ's ability to produce privacy-preserving code which runs significantly faster than previously possible.

High-level goals: ZØ aims to provide an attractive combination of high-level goals of *privacy*, *integrity*, *expressiveness*, and *performance*. While the first two goals are achieved through the use of zero-knowledge, to support ease of programming and expressiveness, ZØ accepts (a subset of) C#, a widely-used general purpose language as input that can run in many settings. Of course, we are not tied to C# and could support another high-level language such as JavaScript, Java, or C++. Our use of a general-purpose language allows developers to include hundreds or thousands of lines of C# or other .NET code, allowing the construction of full-featured GUI-based distributed applications. To enable distributed programming wherever .NET code can run, ZØ supports automatic tier-splitting, inspired by distributing compilers such as GWT [24] and Volta [29]. We primarily target client-server computations (two tiers), although other options such as P2P are also supported by ZØ. Code produced by ZØ can be run on desktops, in the cloud, on mobile devices (Windows Phone) and on the web (Silverlight).

Applications: Much of the inspiration for ZØ came from our desire to be able to use ZKPK techniques to build applications directly analogous to some widely-deployed commercial products, as opposed to toy benchmarks. In our studies detailed in Section 6, we show how they can be (re-)built in a privacy- and integrity-preserving way. For example, our FitBit study was inspired by wireless activity tracking devices manufactured by FitBit (`fitbit.com`) and Earndit (`earndit.com`). The Slice study was inspired by purchase tracking software from Slice, Inc. (`slice.com`). The study Waze app was inspired by Waze, a popular mobile crowd-sourced traffic and real-time traffic and directions software (`waze.com`).

Contributions: To sum up, this paper makes these contributions:

- This paper proposes ZØ, a distributing compiler that allows developers to create highly performant, large distributed applications, while preserving both privacy and integrity.
- ZØ uses precisely calibrated *cost models* to choose which underlying zero-knowledge back-end to employ. Based on the cost model, ZØ statically determines the appropriate *splitting perimeter* for the application to achieve best performance and rewrites it to be run on multiple tiers.
- ZØ is designed to be easily accessible to a regular developer; to this end, we expose zero-knowledge functionality via LINQ, language-integrated-queries. We demonstrate the expressiveness of the ZØ approach by developing 6 case studies directly inspired by commercial applications, ranging from personal fitness tracking (Fitbit) to crowd-sourced traffic-based routing (Waze), to personalized shopping scenarios.
- Using a combination of cryptographic primitives on a set of applications mimicking functionality from existing mobile and web applications, we demonstrate the viability of ZØ: it produces code that can scale to large data inputs and thousands of users in a distributed setting. We evaluate ZØ, focusing on latency and throughput of zero-knowledge tasks. Our cost-fitting models provide an excellent match with the observed performance, with R^2 scores between .97 and .99. Our global optimizer is fast, completing in under 3 seconds on all programs. ZØ produces code that archives as much as 58× speedups compared to state-of-the art zero-knowledge systems. We also find that ZØ is able to effectively optimize across tiers in a distributed application: while the code it generates may be slower on one tier (we observed one case that was 4× slower for the server), the savings at other tiers is always much greater (the same case was 16× faster on the clients).

Paper Organization: The rest of the paper is organized as follows. Section 2 provides motivating examples and some background on zero-knowledge. Section 3 gives an overview of the ZØ approach. Section 5 describes the ZØ compiler implementation. Section 4 talks about both local and global optimizations ZØ performs. Section 7 describes our experimental evaluation. Related work is discussed in Section 8. Finally, Section 9 concludes.

2 Background

To explain the goals of ZØ concretely, we will demonstrate its functionality on a smartphone application with conflicting privacy and integrity needs.

2.1 Example: Retail Loyalty Card

Figure 1 shows the ZØ code for a personalized retail loyalty card mobile app, with functionality similar to Safeway's "Just for U" application or Walgreens' iOS application(Figure 3). Figure 2 shows the end-to-end architecture of this application. Each time the customer reaches the check-out line, this application interacts with the retail terminal in a bi-directional exchange of information. The exchange takes place using the phone's built-in NFC sensor.

First, the application sends a *discount claim* to the retail terminal, pertaining to the items the customer is about to purchase. This discount is computed based on the customer's previous purchases, using personalization to provide enhanced value and incentive for the customer. Zero-knowledge proofs are supplied to ensure the privacy of the customer's shopping history, without sacrificing the trustworthiness of their discount claim.

Second, the terminal sends a list of purchases to the client, corresponding to the current check-out transaction. This list, along with the customer's other

```
public class LoyaltyCard : DistributedRuntime
{
  // Local variable declarations
  [Location(Client)] IEnumerable<int> shophist;
  [Location(Client)] IEnumerable<int> items;
  IEnumerable<Triple> automata;
  IEnumerable<Pair> transducer;

  public void Initialize(string[] args)
  {...}

  public void DoWork(string[] args)
  {
    var discount =
      GetDiscounts(shophist, items,
                   automata, transducer);
    ApplyDiscount(discount);
  }

  [Location(Client)]
  IEnumerable<Pair> GetDiscounts(
   [MaxSize(Purchases)] IEnumerable<int> history,
   [MaxSize(Items)] IEnumerable<int> items,
   [MaxSize(Edges)] IEnumerable<Triple> automata,
   [MaxSize(States)] IEnumerable<Pair> transducer)
  {
    ZeroKnowledgeBegin();
    // Check that the history is in ascending order
    var historyAscendingCheck = history.Aggregate(
      0,
      (last, curel) => check(last <= curel));
    // Get the "discount state"
    var purch_state = history.Aggregate(
      0,
      (state, purch) =>
        automata.First(
         trans => (trans.fld(1) == state) &&
                  (trans.fld(2) == purch)).
                                  fld(3));
    var discount = history.Aggregate(
      new Pair(purch_state, 0),
      (state, purch) =>
        new Pair(
         // Get the next automata state
         automata.First(
          trans => (trans.fld(1) == state.fld(1))
                   && (trans.fld(2) == purch)).
                   fld(3),
         // Total the current state discount
         state.fld(2) + transducer.First(
           edge => edge.fld(1) == state.fld(1)));
    ZeroKnowledgeEnd();

    return new IEnumerable<Pair>(discount);
  }

  [Location(External)] void ApplyDiscount(...)
  {...}
}
```

Figure 1: Running example application: a personalized retail loyalty card.

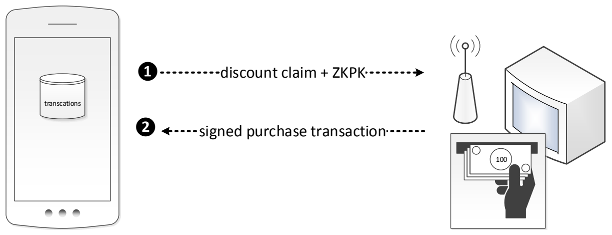


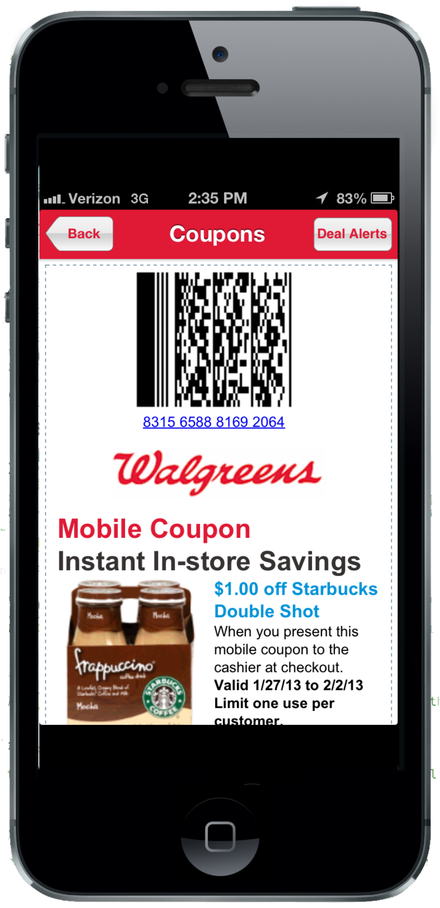
Figure 2: Personalized loyalty card application.

previous purchases, will be stored in a client-side database used to compute a discount the next time the user shops with this retailer.

Application Code: Figure 1 contains C# code for computing the core functionality of this application: using the customer's purchase history to produce a discount, and sending that discount to the retail terminal. It is important to notice that this is standard C#, capable of seamless incorporation into larger bodies of C# code. In fact, ZØ extends on the standard C# compiler, and only applies specialized reasoning to classes that inherit from ZØ's `DistributedRuntime` class. All of the UI and external library code can remain in the application, without affecting the performance and functionality of ZØ. This allows ZØ to scale to large applications with arbitrary legacy dependencies, provided that the sections requiring zero-knowledge reasoning are localized and moderate in size. Several important points bear mentioning.

First, of the four functions, two of them, which we call *worker functions*, contain *location annotations*: `GetDiscounts` is constrained to execute on the client (e.g., the user's smartphone), and `ApplyDiscount` to `External` (e.g., the retail terminal). ZØ generates separate object code for each of these locations, and inserts code to handle the network transfer and data marshalling for any dependencies between these two functions. In order to streamline the code generated by ZØ, the worker functions must always return `IEnumerable` objects, which ZØ's underlying runtime is optimized to quickly marshall and transfer.

Second, the target functionality is computed from the main function `DoWork`, which is called after `Initialize`. `Initialize` gives the application an op-



Mobile Coupon
Instant In-store Savings
$1.00 off Starbucks Double Shot
When you present this mobile coupon to the cashier at checkout.
Valid 1/27/13 to 2/2/13
Limit one use per customer.

Figure 3: Real-world example of the personalized loyalty card application: Walgreens' iOS application.

portunity to prepare the class's local state by reading sensors, buffering data, etc., and can contain arbitrary C# code. `DoWork` is more constrained: it can contain a sequence of calls to worker functions, with no intermediate local computations, branching statements, or loop statements. This allows ZØ to efficiently compute the dependencies between different tiers. In this case, ZØ determines that the return value of `GetDiscounts` (computed on the smartphone) is always used by `ApplyDiscount` (computed on the retail terminal), and inserts code to package and send, or receive and unpack, the necessary data as well as any accompanying zero-knowledge proofs.

Third, the main code is located in `GetDiscounts`, which takes a list of the user's previous purchases (`history`), the user's current check-out items (`items`), and a finite-state transducer (`automata` and `transducer`), and produces a discount dollar value for transfer to the retail terminal. The transducer is produced by the retailer, and is designed to associate past purchases to items that the customer may be interested in buying in the future; the details of designing the transducer are beyond the scope of this work. `GetDiscounts` begins by checking that the purchases are given in ascending order, by their ID numbers; this is a simple optimization that allows the retailer to minimize the size of the transducer. This check is performed by performed using LINQ's `Aggregate` operator, and ZØ's `check` function, which behaves like an assertion. It then proceeds to traverse the transducer's finite-state machine using the customer's shopping history, effectively loading the history into the transducer's memory in preparation for emitting discount values.

Finally, the customer's current items are processed by traversing the finite-state machine, starting in the final state of the previous traversal, and summing the output of the transducer relation. The final sum is returned to `DoWork` as a discount claim.

Zero-knowledge: The entirety of `GetDiscounts` is computed in zero-knowledge, as indicated by the `ZeroKnoweldgeBegin`() and `ZeroKnowledgeEnd`() annotations. Notice that each statement of this method consists of a LINQ query, giving the computation an overall functional form, without using language features such as references, loops, or conditionals. This is necessary to accommodate faithful translation into code that produces zero-knowledge proofs using the zero-knowledge back-ends discussed in Section 2.2. However, the programmer is still able to express zero-knowledge computations in this fragment of standard C#, without dealing with the overhead of inter-language binding between the engines and the main program, and without needing to learn the different input languages understood by each engine.

Finally, a few subtle details of this code bear mentioning. Two of the class variable declarations, `shophist` and `items`, have location annotations that tell ZØ that they should not leave the customer's smartphone without first being processed by zero-knowledge code. This gives the programmer an extra degree of assurance of the code's privacy properties, letting her treat the zero-knowledge code regions like *declassifiers* with additional integrity guarantees. The code in `GetDiscounts` uses `Pair` and `Triple` types, instead of the standard `System.Tuple` <> types supported by the .NET platform. These types are defined by the programmer, and have a restricted form that prevents issues such as side effects or arbitrary method code that might prevent accurate translation into zero-knowledge code. Finally, notice that the parameters to `GetDiscounts` contain `MaxSize` attribute annotations. These optional size annotations allow the ZØ compiler to do precise cost modeling, as explained in Section 4.

2.2 Zero-Knowledge Back-ends

ZØ relies on two zero-knowledge back-ends, Pinocchio [36] and ZQL [19], to produce code that balances privacy and integrity. Each of these back-ends takes an expression, in the form of executable code in a high-level source language, and produces object code that computes the expression over dynamically-provided inputs while building zero-knowledge proofs for the expression on the given input. These engines have very different characteristics that affect performance and usability in different ways, which we outline here.

Pinocchio: Pinocchio utilizes a novel underlying computation model, *Quadratic Arithmetic Polynomials*, to evaluate an expression and produce zero-knowledge proofs [36]. For some computations, it yields performance gains several orders of magnitude beyond previous systems that gave similar functionality, producing proofs of a constant size *regardless* of the size or structure of the target expression.

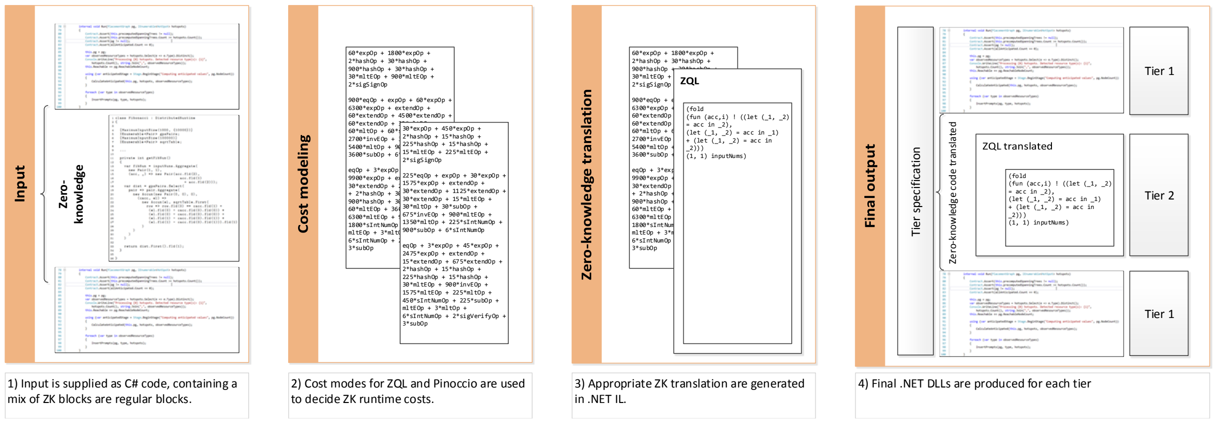
The expression language supported by Pinocchio is a strict subset of C, and the object created for evaluation is an *arithmetic circuit* [36]. An arithmetic circuit is structured identically to a traditional Boolean circuit, but the gates correspond to addition and multiplication, and the wires carry values from a field. The fact that the target circuit must be finite, and cannot encode *side-effects*, imposes necessary conditions on the parts of C that are available. Loops and conditionals are "unrolled" during compilation, so all loops must have static bounds. Likewise, pointers and array indices must be compile-time constants, or simple loop variables (as these are unrolled), thus simplifying cost modeling. Structured data types and function definitions are supported, but the only scalar type allowed is int.

ZQL: ZQL utilizes several fairly recent advances in the theory of zero-knowledge proofs to produce efficient verified private code that operates over functional lists [19]. The underlying cryptographic machinery used by ZQL is more traditional than that of Pinocchio, relying heavily on homomorphic commitment schemes to provide its guarantees. Additionally, the compiler produces code that can be verified using an advanced type checker such as F7 or F*.

The expression language supported by ZQL is a simple functional language without side effects, and limited operator support. In a nutshell, ZQL supports map and fold operations, as well as find operations over tuples of integers. Boolean expressions can only be used inside of find operations, and are currently limited to conjunctions of equality tests; all forms of inequality are not explicitly supported, although the authors plan to support these operations in future versions. In terms of arithmetic, addition, subtraction, and multiplication are supported. Finally, multiple operations can be sequenced using classic functional let bindings. Although these constructs might seem modest at first blush, the ability to perform table lookups using find allows for the evaluation of logic gates, and the list-based map and fold operations place no upper-bound on the size of the program's input, as in the case of Pinocchio. Thus, ZQL offers a powerful language for moderately complex operations over large data.

3 Overview

ZØ allows developers to write code for distributed computation environments that provides guarantees on the *privacy* and *integrity* of their application's data. The privacy guarantees supported by ZØ spec-


Input
Zero-knowledge
Cost modeling
Zero-knowledge translation
ZQL
Final output
Tier specification
Zero-knowledge code translated
ZQL translated
Tier 1
Tier 2
Tier 1
1) Input is supplied as C# code, containing a mix of ZK blocks are regular blocks.
2) Cost modes for ZQL and Pinoccio are used to decide ZK runtime costs.
3) Appropriate ZK translation are generated in .NET IL.
4) Final .NET DLLs are produced for each tier

```
30*expOp + 450*expOp +
2*hashOp + 15*hashOp +
225*hashOp + 15*hashOp +
15*mltEOp + 225*mltEOp +
2*sigSignOp

225*eqOp + expOp + 30*expOp +
1575*expOp + extendOp +
30*extendOp + 1125*extendOp +
30*extendOp + 15*mltEOp +
30*mltOp + 30*subOp +
675*invEOp + 900*mltEOp +
1350*mltOp + 225*sIntNumOp +
900*subOp + 6*sIntNumOp

eqOp + 3*expOp + 45*expOp +
2475*expOp + extendOp +
15*extendOp + 675*extendOp +
2*hashOp + 15*hashOp +
225*hashOp + 15*hashOp +
30*mltEOp + 900*invEOp +
1575*mltEOp + 225*mltOp +
450*sIntNumOp + 225*subOp +
mltEOp + 3*mltOp +
6*sIntNumOp + 2*sigVerifyOp +
3*subOp
```

```
(fold
(fun (acc,i) ! ((let (_1, _2)
= acc in _2),
(let (_1, _2) = acc in _1)
+ (let (_1, _2) = acc in
_2)))
(1, 1) inputNums)
```

Figure 4: ZØ architecture.

ify which parties may view certain data elements used in parts of the application, whereas the integrity guarantees specify assurances on the types of computations and transformations performed on the data elements. We primarily target client-server computations (two tiers), although other options such as P2P are also possible. *Localized computation* [20, 15] is the primary mechanism used to provide privacy guarantees: any computations over private data are constrained to be carried out only by the party to which the data is considered private. "Outsourcing" of computation over private data is avoided at all costs, as there is no efficient mechanism that allows the data inputs of such a computation to remain private.

In order to support a broad range of functionality, it is often necessary for a party to share the output of a computation over private data with an *external* party. For many applications, this introduces a *trust* problem between the party concerned about private data and the party that needs the results of a useful computation over that data: without further assurances, the latter party must simply trust that the former carried out her computations honestly, and that the *utility* of the shared result is sufficient. ZØ addresses this problem by providing an integrity guarantee using zero-knowledge proofs of knowledge [6]. The *zero-knowledge* property of this mechanism affords an opportunity to balance the need for integrity with that of privacy, so that the former party does not need to concern herself with the latter knowing the content of her private data in order to trust the result of her localized computation.

The guarantees provided by ZØ are strong in the sense that they preclude cheating and unwanted snooping on private data by any party taking part in a distributed computation. However, they do not address the higher-level concern of *inferential privacy*, wherein the results of a private localized computation leak additional, unwanted information about the private data source. For example, ZØ might compile a function that translates a sentence from one language to another into a routine that provides a zero-knowledge proof that the translation is performed correctly for each translated sentence. While other parties are not provided with the input sentence because the translation was performed locally by the original party, and the zero-knowledge proof will not reveal anything *explicitly* about the input sentence, the translated sentence will undoubtedly leak a substantial amount of information about the original input data. ZØ does not provide any facilities for preventing or identifying these situations; this is a topic of ongoing research [17] that is orthogonal to the work presented in this paper.

Architecture of ZØ: Figure 4 shows the architecture of the ZØ compiler. The developer provides as input a set of C# source files, which may include arbitrary regions of legacy and library code as well as functionality targeted towards zero-knowledge proof generation. ZØ then enters a *cost modeling* stage (depicted in Figure 4), analyzing the zero-knowledge regions, building performance models that characterize the cost of providing zero-knowledge proof generatiion and verification code for each available zero-knowledge back-end. These models take the form of polynomials over the size of the input data to the zero-knowledge region in the original C# application. ZØ then compares the models to determine which zero-knowledge engine the application should use for each C# statement in the zero-knowledge region, and translates the C# code (depicted in the *zero-knowledge translation* stage of Figure 4) into expressions understood by the appropriate zero-knowledge engine. In the *final output* stage (Figure 4), ZØ decides how to split the application across tiers to maximize performance, given privacy annotations as well as relative *costs* for transmitting data and computing at each tier.

This translation yields a separate module which is callable from the original application, either as an *arithmetic circuit* (Pinocchio) or standard .NET bytecode (ZQL). Finally, ZØ partitions the original C# code, along with the zero-knowledge modules compiled in the previous step, into multiple applications to run at each *service tier*. During partitioning, ZØ inserts code to perform communication, synchronization, data marshaling, and zero-knowledge proof transfer in parallel to the original application code. The resulting modules are standard .NET bytecode that can be distributed onto proper tiers and run without the need for additional specialized software.

Optimization & cost models: Even apparently straightforward applications like the personalized loyalty card app discussed in Section 2.1 contain subtle characteristics that might make zero-knowledge proof generation expensive. It is often the case that one zero-knowledge engine offers significantly better performance for a particular statement, and selecting the appropriate engine for each computation in the zero-knowledge region means the difference between a scalable, low-latency implementation and one that requires hours or days to execute.

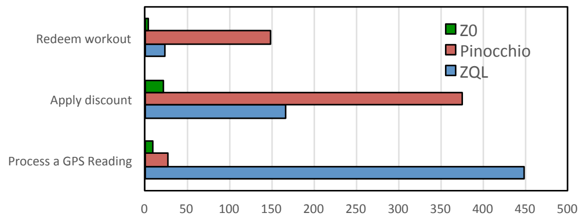


Figure 5: Comparison of times for several applications.

For the loyalty card application in Figure 1, it turns out that the inequality comparisons are better handled by Pinocchio, whereas the table lookups needed to execute the transducer are very inexpensive when performed by ZQL. A comparison of the times to perform the operation on the y-axis for several applications from Section 6 is shown in Figure 5. We can see dramatic differences in performance between the back-ends, with the ZØ approach out-performing either of the two back-ends. ZØ addresses these performance differences by building detailed performance models for each statement in the zero-knowledge region.

In order to achieve complete and accurate results, we exploit several key characteristics of the underlying execution model of each zero-knowledge engine. First, unbounded recursion is not allowed in either engine, and branching behavior has no effect on which statements are executed. This means that the looping behavior of each piece of zero-knowledge proof-generating code is a function only of the size of the input provided to that code. Second, Pinocchio requires static (but arbitrary) upper bounds on the size of each input.

ZØ builds a polynomial over the input sizes for each statement in the zero-knowledge region, and uses these static upper bounds wherever available to evaluate each polynomial, thus arriving at an estimated cost for the statement in two hypothetical compilation scenarios. These cost models are combined using 0-1 integer linear programming techniques, along with placement constraints stemming from privacy requirements, to produce a global split of the application. We discuss our cost models further in Section 4.

Distributed configuration: To support a variety of distributed scenarios, ZØ allows the developer to place code on several different tiers, which are specified using the following *tier labels*: Client (end-user's primary device), External (provider's servers), ClientShare (peer-to-peer nodes), and ClientResource (additional hosts owned by end-user). Tiers impose data confidentiality and integrity constraints, as ZØ makes assumptions about the *trust relationships* between tiers. Data marked as private to Client is entrusted only to ClientResource hosts, but should be kept confidential from External and ClientShare hosts. An identical set of relations holds for data private to ClientResource, with the roles of Client and ClientResource swapped. Data marked private to External is not entrusted to any other tier, and data marked ClientShare only to Client and ClientShare.

The figure in this paragraph shows these relationships; white cells indicate trust, and gray the opposite. At compile time, the user can modify the configuration by specifying *weights* on each tier label indicating the relative cost of computation at that tier, as well as the cost of communication between tiers. ZØ uses these weights during optimization to determine the best placement of code and data amongst the tiers. Data privacy constraints are marked by the programmer by marking certain variables as *private* to a particular tier using the attribute [Private(T_L)], where T_L specifies the tier to which the data is considered private (e.g., Client, External, ...).

	C	**CS**	**CR**	**E**
C	□	▨	□	▨
CS	□	□	□	▨
CR	□	▨	□	▨
E	▨	▨	▨	□

Note that by design, these annotations are very sparse and lightweight: privacy annotations are only needed on (the few) variables that must be kept confidential. Most can be declared without any annotations at all.

Key Generation (prover)	*Add* $\times$ (*CircuitDegree* $\times$ (3 $\times$ *CircuitSize* + 7) + 4 $\times$ *CircuitSize* + 2) + 8 $\times$ *AssignRandom* + ...
Computation (prover)	*CircuitDegree* $\times$ *InterpolationCoef* $\times$ (*Add* + *Mul*) $\times \log^2$(*CircuitDegree*) + ...
Verification (verifier)	*ExpMulB* $\times$ *NInputs* + 12 $\times$ *Pair* + *VerifyConst*

Figure 6: Static performance models for Pinocchio circuits, truncated to fit paper dimensions (full models available in the technical report). *NInputs* represents the number of input wires, *CircuitDegree* the number of multiplication gates, and *CircuitSize* = *NInputs* + *CircuitDegree*.

When ZØ compiles the application and runs a global optimization described in Section 4.5 to place each worker method on a specific tier, privacy annotations are used in part to determine on which tiers a method may reside. These constraints are *hard*, meaning that a privacy annotation that requires a less performant compilation configuration will always be respected; if the privacy constraints conflict with each other, then compilation will not terminate early. Privacy annotations are propagated transitively using a local dataflow analysis [1], so that a variable which depends on a value private to a particular tier is also private to that tier (see Section 4.5.1 for details).

Threat model: Because of its reliance on zero-knowledge back-ends, ZØ makes all of the assumptions needed for security by ZQL [19] and Pinocchio [36]. The result of ZØ compilation will be executed on one or more tiers. We assume that communications between tiers are encrypted, to eliminate man-in-the-middle possibilities, leading to both privacy and integrity violations. Both public key and asymmetric cryptography are reasonable options, depending on the availability of keys. We assume that tiers cannot learn information by means other than direct communication, i.e. Server cannot obtain the list of purchases through side channels, for instance, unless it is directly shared by Client. Our applications that use secret sharing (Waze and Slice in Section 6) also assume that P2P clients do not collude.

4 Cost Models & Optimizations

As outlined in Section 3, in many cases one zero-knowledge engine will outperform the other on a particular computation by a significant factor, giving ZØ

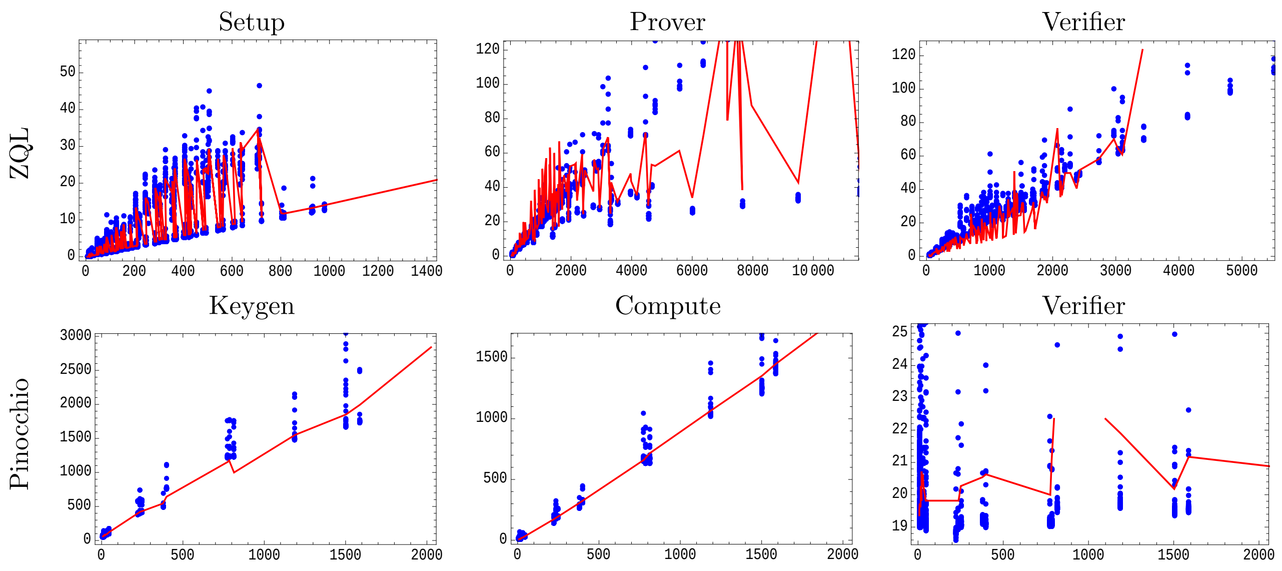


Figure 7: Regression curves and observed data points cost model fitting. Each horizontal axis corresponds to total input size, and each vertical axis to execution time in seconds.

a key opportunity to optimize the code it produces. ZØ optimizes zero-knowledge regions by building detailed performance models that characterize the cost of building and verifying zero-knowledge proofs in each engine. We are able to accomplish this with reasonable accuracy because the execution depth of zero-knowledge regions is statically-bounded (a necessary condition imposed by the underlying engines), and the evaluation of zero-knowledge code universally relies on a few primitive operations. This allows ZØ to build static *cost models* as polynomials over the number of primitive operations each region must execute.

4.1 ZQL: Models from Symbolic Execution

In order to build cost models for ZQL code, we execute the F# "object code" generated by ZQL's compiler *symbolically*, accumulating terms on a polynomial or each operation listed in the F# object code. Symbolic data is represented by polynomials that characterize the size of the corresponding concrete data, or structured sets of polynomials in the case of structured data types. Symbolic integer data is represented by a single polynomial, and symbolic structured data is represented recursively: a symbolic tuple is a tuple of polynomials (having the same width as its concrete counterpart), whereas a symbolic list is a pair, containing a polynomial representing its length and a symbolic value representing the size of each element in the list. For example, the `transducer` list from Figure 1 would be represented by the symbolic value:

```
(transducer_length, (int_size, int_size))
```

Note that although ZQL's integers have varying sizes depending on their magnitude, we introduce a necessary static approximation, and assume here that each integer in the `transducer` list has roughly the same size.

Each primitive operation generated by the ZQL compiler is re-defined to accept symbolic operands, and produce symbolic results that characterize the size of the data computed by that operation. Additionally, an *environment accumulator* is updated to reflect the cost of executing that operation. The environment accumulator is a symbolic integer that represents the current end-to-end cost of execution at all points during the computation. For example, the chained hash operation is defined symbolically as:

```
let hash p1 p2 =
    accumulate (var "hashOp")
    var "lhash"
```

The environment accumulator is updated to reflect the cost of a hash operation with a call to `accumulate`, and a new polynomial representing the size of a hash digest is returned, `var "lhash"`. The operands are ignored, as we assume a constant cost across all hash operations, and the size of the result is not affected by the sizes of the inputs.

Operations that loop over list-valued operands introduce slightly more complexity. The symbolic engine must reflect that the operations performed on individual list elements are executed to the size of the list, but must also be careful to handle the possibility of nested list operations. To accomplish this, a stack is introduced to hold the environment accumulator at each level of loop nesting. When a new loop operation is entered, a fresh accumulator (representing the zero constant) is pushed onto the stack, and each operation within the loop contributes to its value. When the loop is finished executing, this accumulator is popped from the stack, multiplied by the size of the original list operand, and the result is added to the accumulator currently at the top of the stack.

The environment accumulator is inspected at the end of symbolic execution to determine the end-to-end cost of executing the code. Note that the polynomial will contain variables that represent the size of the original input data, as well as variables that represent the cost of primitive operations. For example, the code:

```
let _ = map (fun x -> x + 1) input
```

will produce the polynomial:

```
input_length * add_op
```

We treat the variable corresponding to input length (`input_length`) as free (unbound), and replace the variable corresponding to the cost of addition (`add_op`) with a constant generated using regression.

Because ZQL does not generate branching code, and the execution depth of each loop is always a function of the size of the computation's input data, the models produced by this symbolic execution are accurate. This observation was pointed out by ZQL's authors, and our symbolic engine builds from an incomplete engine they provided, with extensions to symbolically execute all loop-based operations to arbitrary nesting depth, as well as extensions to tune the symbolic result for more accurate performance results.

4.2 Pinocchio: Models from Circuits

Recall that Pinocchio compiles C code into a circuit, which is evaluated by a specialized runtime to produce and verify zero-knowledge proofs. The Pinocchio runtime executes roughly the same code to evaluate every circuit, varying only on the number of times each operation is executed to handle every element of each input list and every operation in the circuit. The cost of evaluating a particular circuit in zero-knowledge can be computed by examining the runtime evaluator's code, and building polynomials that characterize the number of operations that will execute for a circuit with given characteristics. Thus, the cost models ZØ produces for Pinocchio code are based on static polynomials that represent the number of primitive operations needed to execute a circuit with a given number of I/O wires and multiplication gates.

The static Pinocchio cost polynomials for each stage (*key generation*, *computation*, and *verification*) are given in Figure 6; the models for key generation and computation are truncated for formatting clarity, and are provided to give the reader an intuitive understanding of the form each model takes. The authors of Pinocchio generously provided asymptotic expressions characterizing the cost of each stage, which we then tuned to match the Pinocchio implementation more closely[1]. As with the ZQL models, most of the coefficients correspond to costs of individual primitive operations, e.g. *Add* for field addi-

[1]This was not required for the model of Verification time, whose asymptotic expression as provided by the authors closely matches observed execution times

tion, *AssignRandom* for generating random wire values. The free variables correspond to circuit characteristics, with *NInputs* representing the number of input wires and *CircuitDegree* the number of multiplication gates. *Size* is the sum of these values, and although not an independent input, is used for clarity and convenience to reduce the printed size of the models.

The only component of Pinocchio evaluation whose execution cost might vary among circuits with identical parameters is polynomial interpolation, which is performed in the computation stage (the details of why this is necessary are beyond the scope of this work, Pinocchio [36] for more information). Pinocchio performs this in $O(n \log^2 n)$ time, where n is the circuit degree. Although this may account for some divergence between predicted and actual execution time, we have found the difference to be negligible (see Section 4.4), and this has not prevented us from building useful models of Pinocchio execution costs.

4.3 Modelling Proof Size

As will be explained in Section 4.5.1, the size of the proof produced by each zero-knowledge engine is relevant when deciding how to partition worker methods between tiers. We can provide symbolic expressions for the proof size of ZQL computations using the same technique described in Section 4.1: because each symbolic value represents the size of the corresponding concrete data stored in that value, we obtain a polynomial expression for the proof size by symbolically executing the prover module produced by the ZQL compiler, and summing each polynomial component of the output. Producing a proof size for a Pinocchio computation is trivial: all Pinocchio proofs have the same size (288 bytes) [36].

4.4 Local Optimization

Given a cost model produced using one of the methods described in Section 4.1, we need to instantiate the coefficients corresponding to primitive operation costs before we can compare the costs of selecting each zero-knowledge engine. To derive concrete coefficients, we use regression to match the model to observed execution times.

	% Difference			Absolute difference (s)		
	Setup	**Prover**	**Verif.**	**Setup**	**Prover**	**Verif.**
Waze	26.43	10.17	2.18	0.83	0.11	0.05
Slice	8.39	8.91	1.77	0.15	0.96	0.29
FitBit	8.81	12.70	3.84	0.10	2.41	0.73
Loyalty	12.57	12.88	0.37	0.33	0.37	0.01
Average	14.05	11.17	0.35	0.96	0.27	12.84

(a) ZQL

	% Difference			Absolute difference (s)		
	KeyGen	**Prover**	**Verif.**	**KeyGen**	**Prover**	**Verif.**
Waze	10.09	36.10	1.04	0.03	0.00	0.00
Slice	9.20	19.45	0.70	0.02	0.01	0.00
FitBit	25.18	20.36	0.82	0.09	0.01	0.00
Loyalty	34.86	19.41	1.04	1.17	0.46	0.00
Average	19.83	23.83	0.90	0.33	0.12	0.00

(b) Pinocchio

Figure 8: Regression results for each application.

Regression: With the exception of Pinocchio's computation-stage cost model, the cost models produced for both ZQL and Pinocchio are always linear in the coefficients corresponding to primitive operations. This is because we assume, based on our domain knowledge of these primitive operations, that the execution of each operation is independent of the others. While this may oversimplify matters to a small degree due to cache effects and compiler optimizations, incorporating such effects into the models would lead to considerably more complex terms.

Because of this, we use least-squares regression to derive coefficients for all models except those for Pinocchio's compute-stage model. We use Mathematica's implementation of this algorithm, which generally produces answers quickly. To cope with the non-linearity in Pinocchio's compute-stage model, we use the Gauss-Newton method [?] with at most 1,000 iterations and a randomly-chosen starting point. Again, we use Mathematica's implementation of this algorithm for all of our computations.

Cost-fitting results: To derive the necessary coefficients for our models, we built a regression training application in ZØ consisting of several basic operations likely to appear in zero-knowledge applications. The training application takes as input a list of integers, and computes an aggregate sum, scalar product, second-degree polynomial, boolean mapping, and table lookup on the list. We compiled this application to use both all-ZQL and all-Pinocchio zero knowledge computations, and ran it ten times for each zero-knowledge engine using a fixed list size ($n = 100$). We used the techniques described in Section 4.1 to build performance models of this application for each back-end, and performed linear regression to learn coefficients corresponding to the execution time of each primitive operation appearing in the cost model. We then compiled each application described in Section 6 to use either all-ZQL or all-Pinocchio zero-knowledge computations, executed each zero-knowledge region ten times, and recorded the deviation between execution time predicted by the regression-trained cost models and the mean execution time observed over all experiments for a given application.

The results of this analysis are presented in Figures 8 and 9.Figure 8 presents the prediction error of the trained cost models, both as a percentage of the total zero-knowledge execution time as well as in absolute seconds. Note that the performance models derived for ZQL are on the whole more accurate than those for Pinocchio, ranging between 1–20% error on average. The absolute prediction error is quite small, topping out at 1.17 seconds for the Pinocchio key generation routine of the Loyalty application (this accounts for 34.86% of the total time for the application). Figure 9 shows the coefficient of determination (R^2) for each performance model, which are all around 0.99: the models generated by ZØ accurately reflect the structure of the execution time of generated code.

	Setup	Prover	Verifier
ZQL	0.97	0.99	0.99
Pinocchio	1.00	1.00	0.99

Figure 9: R^2 for regression models.

Summary: To summarize, ZØ is able to build performance models of zero-knowledge regions that predict actual execution time within tenths of a second in most cases, which provides ample accuracy to make a correct decision when selecting zero-knowledge engines at compile-time.

4.5 Global Optimization

ZØ builds cost polynomials to characterize the expense of each zero-knowledge operation in the target application. However, selecting the least expensive engine for each operation is oftentimes not as straightforward as evaluating each polynomial at a target input size and choosing the engine corresponding to the lesser value — it may be the case that a less expensive operation on the prover's side requires a more expensive operation on the verifier's side, and depending on the application computation may be more expensive for the verifier. Alternatively, there may be several ways to partition an application between tiers while preserving the privacy of variables at each tier, with each partition yielding a different trade-off between computation and communication cost. To address these concerns, ZØ performs *global optimization* on the application to balance the cost of computation and communication among differentiated tiers.

4.5.1 Solving Global Optimization

The rules for performing global optimization are given in Figure 10. The top portion of Figure 10 specifies the inference rules needed to generate the privacy and functionality constraints, and the bottom portion the objective function used to characterize the suitability of a solution to the constraints. The rules are applied as part of a traversal of the program's abstract syntax tree. Each inference rule either updates the set of constraints collected for a program, or a *context* Γ; they are of the form:

$$\frac{Antecedent}{\Gamma \vdash C, Pattern \Rightarrow C'} \qquad \frac{Antecedent}{\Gamma \vdash C, Pattern \Rightarrow \Gamma'}$$

Γ is the *context* of the analysis, and tracks which method the traversal is currently in, as well as whether the traversal is in a zero-knowledge region and which external methods have an affinity to a particular tier. C is a set of constraints. *Pattern* is an AST pattern, such as $v = f(v_1, \ldots)$ to match an assignment. *Antecedent* is a pre-condition for using a rule: whenever *Antecedent* is true, then either the set of constraints C is updated to a new set C', or

$$\frac{}{\Gamma \vdash C, [\texttt{Private}(\mathit{Loc})]\ \texttt{Type v} \Rightarrow C \leftarrow C \cup p_v = \phi(\mathit{Loc})}$$

$$\frac{}{\Gamma \vdash C, \texttt{Type MethodName } (...)\{...\} \Rightarrow \Gamma \leftarrow \Gamma[\textsf{CurMethod} \mapsto \texttt{MethodName}]} \qquad \frac{\Gamma(\textsf{ZKRegion}) = \textsf{False}}{\Gamma \vdash C, v_1 = v_2 \Rightarrow C \leftarrow C \cup p_{v_1} = p_{v_2}}$$

$$\frac{}{\Gamma \vdash C, \texttt{ZKBegin}() \Rightarrow \Gamma \leftarrow \Gamma[\textsf{ZKRegion} \mapsto \textsf{True}]} \qquad \frac{}{\Gamma \vdash C, \texttt{ZKEnd}() \Rightarrow \Gamma \leftarrow \Gamma[\textsf{ZKRegion} \mapsto \textsf{False}]}$$

$$\frac{\Gamma(\textsf{ZKRegion}) = \textsf{False} \quad f \text{ is a worker method} \quad v_i \text{ is defined by return value of } f_i}{\Gamma \vdash C, v = f(v_1, \ldots, v_n) \Rightarrow C \leftarrow C \cup \left(\bigwedge_{1 \le i \le n} (p_{v_i} = p_f \vee p_{v_i} = \phi(\mathit{Any})) \wedge (p_f \ne pf_i \implies c_{f_i, f} = 1)\right)}$$

$$\frac{\Gamma(\textsf{ZKRegion}) = \textsf{False} \quad f \text{ is an external method} \quad \Gamma(f.\texttt{ExecutionReq}) = l_{exec} \quad \Gamma(\textsf{CurMethod}) = f}{\Gamma \vdash C, v = f(v_1, \ldots, v_n) \Rightarrow C \leftarrow C \cup p_v = p_f \wedge p_f = l_{exec}}$$

$$\frac{\Gamma(\textsf{ZKRegion}) = \textsf{False}}{\Gamma \vdash C, v = \texttt{new}\ f(v_1, \ldots, v_n) \Rightarrow C \leftarrow C \cup \bigwedge_{1 \le i \le n} p_v = p_{v_i} \vee p_{v_i} = \phi(\mathit{Any})}$$

$$\frac{\Gamma(\textsf{ZKRegion}) = \textsf{True}}{\Gamma \vdash C, v = \star \Rightarrow C \leftarrow C \cup p_v = \phi(\mathit{Any}) \wedge z_v \in \{0,1\} \wedge q_v \in \{0,1\} \wedge z_v = 1 \oplus q_v = 1} \qquad \frac{\Gamma(\textsf{ZKRegion}) = \textsf{False}}{\Gamma \vdash C, v = \star \Rightarrow C \leftarrow C \cup \wedge z_v = 0 \wedge q_v = 0}$$

$$\frac{\Gamma(\textsf{CurMethod}) = f}{\Gamma \vdash C, v \text{ is used} \Rightarrow C \leftarrow C \cup p_v = \phi(\mathit{Any}) \vee p_f = p_v}$$

$$\begin{aligned}
\mathit{Minimize} \quad & \sum_{f_i, f_j \in \text{Methods}} c_{f_i, f_j} \text{ComCost}(p_{f_i}, p_{f_j}) \text{DataSize}(f_i) \\
+ \quad & \sum_{v \in \text{Variables}} z_v (\text{ZQLProver}(v) \text{TierComputeCost}(v, \mathit{Prover}) + \text{ZQLVerify}(v) \text{TierComputeCost}(v, \mathit{Verifier})) \\
+ \quad & \sum_{v \in \text{Variables}} z_v (\text{PinocchioProver}(v) \text{TierComputeCost}(v, \mathit{Prover}) + \text{PinocchioVerify}(v) \text{TierComputeCost}(v, \mathit{Verifier}))
\end{aligned}$$

Figure 10: Global optimization constraint generation and objective rules.

Γ is updated to a new context Γ', according to the specifics of the rule.

The rules in Figure 10 use a variable p_v for each program variable v to indicate the privacy level of v; levels correspond to integers that are mapped to tiers by the function ϕ. ϕ maps the tier *Any* (corresponding to the constraint that a variable may appear on any tier) to the value 0, and all other locations to positive integers. Similarly, a variable p_f is created to track the execution location of each worker method f. The constraints also create a variable c_{f_1,f_2} for each pair of worker methods f_1 and f_2. c_{f_1,f_2} takes the value 0, except whenever there is a *tier crossing* between f_1 and f_2, meaning f_2 uses a value computed by f_1, and f_1 and f_2 do not reside on the same tier, in which case it takes the value 1. Finally, each program variable v is associated with two additional constraint variables z_v and q_v, corresponding to whether the zero-knowledge proof of v is computed by ZQL (in which case $z_v = 1$) or Pinocchio (in which case $q_v = 1$). z_v and q_v are mutually exclusive, i.e. $z_v = 1 \oplus q_v = 1$, as the proof of each variable is computed by at most one zero-knowledge engine. If v is not defined inside of a zero-knowledge region, then $z_v = q_v = 0$.

The rules propagate privacy concerns among the variables in a straightforward fashion: for an assignment $v = f(v_1, \ldots, v_n)$ or $v = v_1\ op\ v_2\ op\ \ldots\ op\ v_n$, the constraints are updated to reflect that either $p_v = p_{v_i}$ or $p_{v_i} = \phi(\mathit{Any})$, for all v_i on the right-hand side of the assignment. Intuitively, either v has the same privacy requirements as v_i, or v_i does not have any privacy requirements at all. Similarly, whenever a variable v is referenced in a worker method f, either f must be placed at the tier matching the privacy requirement of v, or v must have no privacy requirement.

Whenever v is assigned in a zero-knowledge region, we constrain its privacy requirement to *Any*, effectively *declassifying* v. Whenever v is the target of an assignment whose right-hand side invokes external code, we assume that v must remain private to the tier on which its host worker method executes. This is a conservative over-approximation, based on the possibility that external code can perform arbitrary actions outside the purview of ZØ's analysis capabilities, such as leaking sensitive files or memory into return values.

The objective function in Figure 10 can be understood in two parts. The first part corresponds to the communication cost of any tier crossings in the tier partition: for each pair of worker methods f_i, f_j, the crossing variable c_{f_i,f_j} is multiplied by the cost of sending data between the tiers $\text{ComCost}(p_{f_i}, p_{f_j})$ and the size of the proofs that need to be communicated between the functions. The proof size is computed using cost models, as described in Section 4.1. Recall that c_{f_i,f_j} is zero except in solutions where f_i and f_j are placed on different tiers.

The second part of the objective function corresponds to the cost of building and verifying zero-knowledge proofs using the engines selected by the current solution. For each variable, there is a term corresponding to its proof generation and verification cost for each engine, multiplied by the cost of computation at the corresponding tier. As with the tier crossing variables c_{f_1,f_2}, z_v and p_v are zero except when the solution selects ZQL or Pinocchio for the variable v, respectively, so each term will only contribute to the final cost when the solution selects a particular zero-knowledge engine.

Performance of global optimization: We implemented our global optimization algorithm as part of the ZØ compiler. CCI2 is used to traverse the AST of the target code, and our cost modeler from Section 4.1 is used to generate the objective function.

To perform the constrained optimization needed to find an optimal solution, we used the Nelder-Mead method [37] with at most 100 iterations. We looked for integer solutions over the full space of possible valuations of p_v, z_v, q_v, and c_{f_i,f_j}.

We ran the global optimization algorithm presented in Section 4.5.1 on each of our applications to determine the amount of time needed to find an optimal solution.

The results are presented in Figure 11. Each application resulted in between 30 and 300 constraints, and the constraint solver found an optimal solution in under three seconds for all applications. Because Nelder-Mead is an approximate numerical optimization algorithm, it is possible that it would return a *local* minimum.

However, we checked the solution returned for each application, and verified that it corresponded to the true global minimum. Figure 12 shows examples of ZØ-computed global splits for two representative applications.

	Constr.	**Time**
FitBit	179	1.50
Waze	263	2.65
Sice	230	2.14
Loyalty	38	0.01
NIDS	48	0.18

Figure 11: Global optimization performance, showing solver time in seconds for the benchmarks in Section 6.

5 Implementation

At the core of ZØ is a compiler that translates C# code into an application that executes on multiple tiers, and builds zero-knowledge proofs where needed to provide privacy and integrity on the inputs to the application. In order to make privacy analysis, zero-knowledge translation, and aggressive optimization feasible, ZØ supports a subset of C# that includes certain LINQ (language integrated queries [38]) functionality and support for external code. To ensure that the external code does not interfere with the privacy, integrity, and optimization goals of ZØ, the contexts in which it is allowed are limited in some cases. In this section, we describe the subset of C# that is supported by ZØ, as well as the tier-splitting and zero-knowledge translation algorithms implemented in the compiler.

5.1 What Do We Support?

The ZØ compiler works directly on .NET assemblies (CIL DLLs) as input. While in principal this means that it could work on code compiled from any .NET language, in practice the tier-splitting and

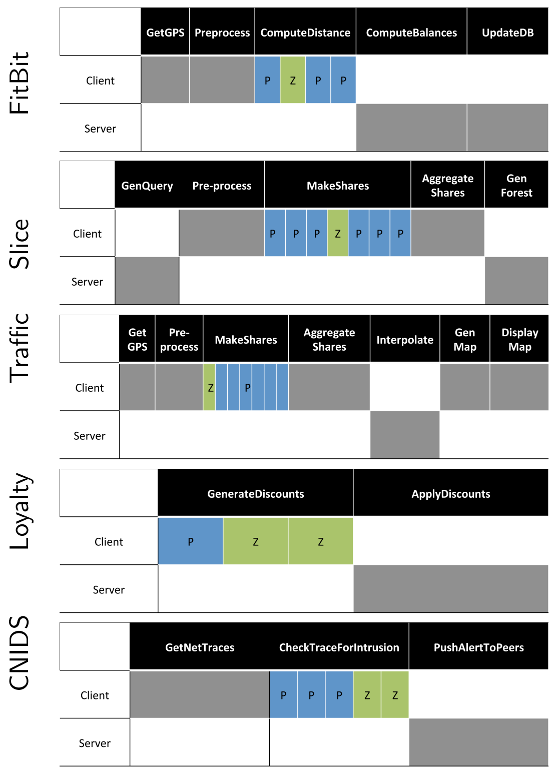


Figure 12: Splits produced by global ZØ optimizations. Grey cells indicate computation location, blue cells Pinocchio computations, and green cells ZQL computations.

Main program definition

Program	::=	InitBlock MainBlock MethodDef* TypeDef*
InitBlock	::=	CSMethodSig VarDecl*
MainBlock	::=	CSMethodSig WorkerStmt$^+$
MethodDef	::=	CSMethodSig (ExternCall | LinqStmt)$^+$
TypeDef	::=	`class` Id { CSFieldDef + }
CSMethodSig	::=	PrivacyAnnot CSType Id(...){ ... }

Statements

WorkerStmt	::=	SleepStmt | CallStmt | ZKAnnot
SleepStmt	::=	`WorkerSleep`(Integer, Integer, Integer)
CallStmt	::=	(Id =)? MethodCall
ExternCall	::=	`return External`.Id "(" Id* ")"
LinqStmt	::=	(Id =)? LinqExpr
VarDecl	::=	(PrivacyAnnot | SizeAnnot)? Id(= CSExpr)?

Expressions

Lambda	::=	"(" Id* ")" ⇒ LambdaExpr
LambdaExpr	::=	MethodCall | ArithOrBoolExpr | FieldExpr | NewObj
LinqExpr	::=	LambdaLinqExpr | ZipLinqExpr
LambdaLinqExpr	::=	Id.LambdaLinqId(Lambda)
LambdaLinqId	::=	Select | Aggregate | First
ZipLinqExpr	::=	Id.`Zip`(Id, NewAnonObj)
MethodCall	::=	Id "(" LambdaExpr* ")"
NewObj	::=	NewAnonObj | NewStaticObj
NewAnonObj	::=	`new` {(Id = LambdaExpr)$^+$}
NewStaticObj	::=	`new` MethodCall
FieldExpr	::=	Id.`fld`⟨Type⟩(Int)

Annotations

ZKAnnotat	::=	ZeroKnowledgeBegin() | ZeroKnowledgeEnd()
PrivacyAnnot	::=	[Private(T_L)]
SizeAnnot	::=	[MaximumInputSize(Int$^+$)]

Figure 13: BNF syntax for the subset of C# supported by ZØ. Entities prefixed with CS correspond to the corresponding C# syntax entity.

zero-knowledge conversion facilities in the compiler are designed to work on a subset of C# with LINQ. The syntax accepted by ZØ is summarized in Figure 13.

Program structure: The main program is structured into three parts: an initialization routine (InitBlock, contained in a method `Initialize`), the main body (MainBlock, contained in a method `DoWork`), and the worker methods (MethodDef). The initialization routine may consist of a sequence of arbitrary C# assignment statements, including calls to methods in external libraries not written in ZØ's input language. The main block consists of a sequence of method calls, assignment statements, and sleep statements. Each method call in the main body must be to a worker method defined in the ZØ application.

Zero-knowledge regions: The body of each worker method can contain calls to external methods, standard C# arithmetic and Boolean operations, and a subset of the standard LINQ data processing operations. Regions comprised of LINQ operations can be converted into zero-knowledge proof-generating object code using either available zero-knowledge engine (ZQL or Pinocchio). The supported LINQ operations include Select, Aggregate, First, and Zip. Select provides the ability to project the data in one list into a new list, while performing arithmetic and Boolean operations on each item in the original source list. Aggregate provides the ability to compute iterated functions over a list, maintaining an order-sensitive state through the iteration, which is eventually returned as the result of the operation. First provides the ability to perform searches over lists, using a programmer-defined predicate to determine which element of the list to match. Finally, Zip provides the ability to combine multiple lists, applying arithmetic and Boolean operations to each pair of items from the original source lists.

Zero-knowledge regions are specified by the programmer using a pair of methods ZeroKnowledgeBegin and ZeroKnowledgeEnd. Because zero-knowledge computations provide both integrity and privacy, these annotations serve a dual purpose. First, the programmer is denoting that the variables which are *live* [1] at the end of a zero-knowledge region are trusted across all tiers: the values have accompanying proofs that any tier can examine to verify that the computations in the zero-knowledge region are performed correctly. Second, these regions serve to *declassify* private values that are used as inputs to a zero-knowledge region; this is in line with the approach taken by ZQL [19]. Because the inputs to zero-knowledge regions are kept private, except in cases where the computations are in some way invertible, the output values that depend on these inputs are considered public to all tiers.

Formal reasoning about composing proofs obtained from different zero-knowledge back-ends remains an interesting avenue for future work. Because this work involves experimentation with bleeding-edge cryptographic tools, there does not exist a readily-available composition theorem that would support reasoning about Pinocchio and ZQL.

Note that ZØ treats external methods invoked by worker methods as *opaque black boxes*. To ensure safety, we must assume that external code can cause arbitrary side effects such as reading or writing private data to persistent storage and communicating over the network. The only assumption ZØ makes is that external methods to not overwrite the active memory of the running ZØ Application Domain, for which we rely on the underlying .NET runtime to enforce. For this reason, code appearing in zero-knowledge regions cannot invoke external functionality. Furthermore, when propagating privacy constraints on variables, we assume that an external method always returns data private to the tier on which it is executed.

ZØ also supports *size annotations* on variable declarations using the attribute [`MaximumSize`(...)]. These are used to help ZØ's optimizer select an appropriate zero-knowledge engine during compilation, and are optional. They are discussed further in Section 5.3.

5.2 Distributing Compiler

ZØ partitions the given target application into code that runs on multiple tiers, inserting marshalling and synchronization code [24, 29] as necessary to ensure that the compiled functionality matches that specified in the original input program. The rewrite process is implemented as a bytecode-to-bytecode transformation within the CCI 2 rewriting framework for .NET [32]. The location of code and data on each tier can either be specified explicitly by the user, as attribute annotations on methods, or it can be inferred by the compiler to optimize computation and communication performance while respecting privacy constraints. In this section, we focus on the process of compiling code to execute seamlessly on multiple tiers, postponing the discussion relating to automatic location inference and optimization for Section 4.5.1. We assume that the target tier for each method is provided as input to the compiler by the optimizer, as described in Section 4.5.

Code partitioning between tiers takes place at method granularity, and data partitioning is determined by the chosen code partition; data is transmitted between tiers on-demand, with all of the data represented by a variable used by a particular method being transmitted at once as it becomes available. Only worker methods can be split between different tiers, so all external code referenced by the application is present on each tier. This allows the compiler to avoid a potentially expensive deep-dependency analysis of the referenced external code, while keeping the dependency analysis of the target application

$$\frac{\Sigma(\mathit{Method}) = \mathbf{L} \quad \mathit{Local} = \{\mathit{arg}_i \mid \Sigma(\mathit{arg}_i) = \mathbf{L}\} \quad \mathit{Remote} = \{(\mathit{loc}, \mathit{arg}_i) \mid \Sigma(\mathit{arg}_i) = \mathit{loc} \wedge \mathit{loc} \neq \mathbf{L}\}}{\mathbf{L} \vdash \Sigma, \mathit{Method}(\mathit{arg}_1, \ldots, \mathit{arg}_n) \Rightarrow \Sigma, \texttt{GetArgsThenCall}(\mathit{Method}, \mathit{Remote}, \mathit{Local})}$$

$$\frac{\Sigma(\mathit{Method}) \neq \mathbf{L} \quad \mathit{LocalDefs} = \{\mathit{arg}_i \mid \Sigma(\mathit{arg}_i) = \mathbf{L}\}}{\mathbf{L} \vdash \Sigma, \mathit{Method}(\mathit{arg}_1, \ldots, \mathit{arg}_n) \Rightarrow \Sigma, \texttt{ExfiltrateData}(\mathit{LocalDefs})} \qquad \frac{}{\mathbf{L} \vdash \Sigma, \texttt{WorkerSleep}(\cdot) \Rightarrow \Sigma, \texttt{WorkerSleep}(\cdot)}$$

$$\frac{\mathbf{L} \vdash \Sigma, \mathit{Method}(\mathit{arg}_1, \ldots, \mathit{arg}_n) \Rightarrow \mathit{expression} \quad \Sigma' = \Sigma[\mathit{var} \mapsto \Sigma(\mathit{Method})]}{\mathbf{L} \vdash \Sigma, \mathit{var} = \mathit{Method}(\mathit{arg}_1, \ldots, \mathit{arg}_n) \Rightarrow \Sigma', \mathit{var} = \mathit{expression}}$$

Figure 14: Transformation rules used by the ZØ compiler.

localized to the main method, `DoWork`.

Runtime support: The architectural principle that guides ZØ's tier-splitting algorithm can be summarized as follows: *whenever possible, delegate the data communication and synchronization operations necessary to support functionality to a runtime API.* Each application compiled by ZØ is linked to a runtime library that provides an API for communicating data and synchronization between separate tiers. When the compiler performs tier splitting, rather than inlining complex code to perform the tasks, simple calls to this API are inserted to perform the "heavy lifting" of tier crossings at runtime. While in general this approach limits flexibility and does not scale to tier crossings that occur at arbitrary granularity, it is a simple and clean design choice that fully supports the limited language supported by core ZØ applications.

The partitioning algorithm used within the main method is rule-driven, as depicted in Figure 14. The rules are invoked once for each tier used in the partition; this state is represented by the symbol **L**. Before the rules are applied, a simple dependence analysis is performed to determine the *root tier* of the data represented by each variable; the root tier corresponds to the tier at which the data for a particular variable is computed. Because the syntax of ZØ's target language does not allow dereferences or explicit loops in the main method, this is an inexpensive def-use dataflow analysis [1]. This information is stored in the context Σ, which is used in each rule to insert data exfiltration and synchronization code whenever tier crossings occur in the code.

The first two rules in Figure 14 correspond to method calls. In the first case, the location of the method matches the location of the current rule application reflected in **L**. In this case, the arguments to the method are divided into two sets, *Local* and *Remote*, where the dependency analysis has determined that the data reflected by the arguments in *Local* is already present on the current tier, whereas the data reflected in *Remote* is not. The compiler replaces such a method call with a call to a runtime library function `GetArgsThenCall`, which uses the information in Σ (which is made available at runtime) to communicate with the tiers currently hosting the data in *Remote* in order to retrieve the necessary data. `GetArgsThenCall` waits for all of the data to arrive, then invokes the original method.

The second rule corresponds to the case when the method does not reside on the same tier as the current rule application **L**. In this case, any arguments needed by the method that currently reside on the tier reflected by **L** are exfiltrated to the tier on which the method executes. This is done by a call to a runtime library function `ExfiltrateData`, which establishes a listening TCP/IP socket and waits for the tier at **L** to connect and request data.

The remaining rules are simpler, and correspond to assignment and `WorkerSleep` statements. In the former case (assignment), the rules are applied recursively to the right-hand side of the assignment to insert any exfiltration and synchronization code. When this is complete, the assignment is inserted with the same left-hand side, and a potentially modified right-hand side. In the latter case (`WorkerSleep`), the statement is emitted unchanged, as the sleep is assumed to take place on all tiers (roughly) simultaneously.

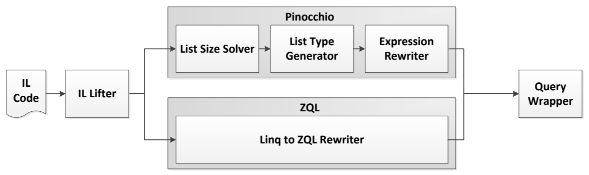


Figure 15: LINQ → translation process.

5.3 Translating LINQ to Zero-Knowledge

In order to satisfy privacy and integrity constraints, our compiler translates some statements containing LinqExpr components in the worker methods into code that generates zero-knowledge proofs of knowledge. To accomplish this, ZØ relies on two *zero-knowledge back-ends*: ZQL [19] and Pinocchio [36]. Each back-end is itself a compiler, accepting as input an expression of a computation, and producing executable code to produce a zero-knowledge proof of the computation for a given set of inputs. As such, each back-end supports its own *expression language* with significantly different characteristics. The challenge addressed in this section is the translation of the common subset of LINQ supported by ZØ into the expression languages of these back-ends.

Figure 15 gives an overview of our back-end compilation process for ZQL and Pinocchio. The details differ significantly for each back-end, converging only on the first and last steps which correspond to lifting low-level intermediate language code into a higher representation and inserting I/O marshaling instructions before and after the compiled object code. This divergence of functionality is necessary given the differences between the two expression languages: ZQL's expression language is essentially a small subset of pure standard ML, whereas Pinocchio's is a subset of C with restrictions on data types and loop bounds. Because the subset of LINQ functions supported by ZØ corresponds to a small core of functional expressions, translating from ZØ to Pinocchio is much more involved than to ZQL.

5.3.1 Pinocchio

The structure of C code is substantially different from the types of Linq queries allowed by ZØ, and Pinocchio's additional restrictions make translation more complicated yet. First, all list sizes used in the Pinocchio expression must be statically-declared, and any operation over a list requires a static value to bound the corresponding loop statement. The LINQ commands in ZØ do not have these restrictions, so we must find a way to derive the needed information. Second, many expression forms in ZØ's LINQ commands have no corresponding expression form in C: they must be converted into statements whose side-effects are available as sub-expressions to enclosing expressions.

To perform translation to Pinocchio, ZØ follows a three-step process. First, static values for the size of each identifier that refers to a list value are derived using a constraint solver. The basis for this computation is a set of annotations provided by the developer, which indicate upper bounds on the sizes of certain input lists.

List Size Resolution: As previously discussed, Pinocchio requires static sizes for all lists and list operations, so our translation procedure requires a mapping from identifiers (for those that refer to list objects) to size constants. To produce such a mapping, we use a constraint resolution procedure over a set of bounding constraints generated by traversing the source expression. The rules for generating the constraints are given in Figure 16. Each rule is of the form Γ, *Syntactic Element* $\Rightarrow \Gamma'$, where Γ and Γ' are sets of constraints. The constraints for each LINQ command are straightforward. The outcome of Select, Aggregate, and Zip operations has the same size as the input variable(s). The outcome of a First statement has the size of the elements contained in the input list.

The rules are invoked by a procedure that traverses each node of the program's AST, and performs syntactic matching on the entity represented by each node and the *Syntactic Element* of each rule. As the traversal proceeds, a list of constraints is maintained, and updated when rules match AST nodes. When the AST traversal completes, he set of constraints gener-

$$
\begin{aligned}
&\mathit{con}(\mathit{expr}) = \\
&\quad \{\mathit{id.elt}\} \quad \textbf{when } \mathit{expr} \text{ is } \mathit{id}.\mathsf{First}(\ldots) \\
&\quad \{\mathit{id}_1, \mathit{id}_2\} \quad \textbf{when } \mathit{expr} \text{ is } \mathit{id}_1.\mathsf{Zip}(\mathit{id}_2, \ldots) \\
&\quad \{\mathit{id}\} \quad \textbf{when } \mathit{expr} \text{ is } \mathit{id}.\mathsf{Aggregate}(\ldots) \\
&\quad \{\mathit{id}\} \quad \textbf{when } \mathit{expr} \text{ is } \mathit{id}.\mathsf{Select}(\ldots) \\
&\quad \{\mathit{id}.n\} \quad \textbf{when } \mathit{expr} \text{ is } \mathit{id}.\mathsf{Fld}(n) \\
&\mathit{con}(\mathit{id}) = \{\mathit{id}\}
\end{aligned}
$$

$$\text{C-FieldDef1}\ \frac{\varphi \le \mathit{id} = \mathtt{x} \wedge \mathit{id.elt} = 1}{\Gamma, [\mathtt{MaximumInputSize(x)}]\ \mathsf{IEnumerable}\langle\mathsf{T}\rangle\ \mathit{id} \Rightarrow \Gamma \cup \{\varphi\}}$$

$$\text{C-FieldDef2}\ \frac{\varphi = \begin{array}{l} \mathit{id} \le \mathtt{x} \wedge \mathit{id.elt} \le n_1 \wedge \mathit{id.elt.elt} \\ \le n_2 \wedge \cdots \wedge \mathit{id}.(\mathit{elt})^k \le n_k \wedge \mathit{id.elt}^{k+1} = 1 \end{array}}{\Gamma, [\mathtt{MaximumInputSize}(\mathtt{x}, \{\mathtt{n}_1, \ldots, \mathtt{n}_\mathtt{k}\})]\ \mathsf{IEnumerable}\langle\mathsf{T}\rangle\ \mathit{id} \Rightarrow \Gamma \cup \{\varphi\}}$$

$$\text{C-Method}\ \frac{\mathit{id}(\mathit{id}_1, \ldots, \mathit{id}_n)\ \text{ is a call site}}{\Gamma, \mathsf{Type}\ \mathit{id}(\mathit{id}_1^f,\ \ldots,\ \mathit{id}_n^f)\ \{\ \ldots\ \} \Rightarrow \Gamma \cup \{\mathtt{id}_1^f \ge \mathtt{id}_1, \ldots, \mathtt{id}_n^f \ge \mathtt{id}_n\}}$$

$$\text{C-New}\ \frac{V_i = \mathit{con}(\mathit{expr}_i)}{\Gamma, \mathtt{new}\ \mathit{id}(\mathit{expr}_1, \ldots, \mathit{expr}_n) \Rightarrow \Gamma \cup \bigcup_{1 \le i \le n}\{\bigwedge_{v \in V_i} \mathit{id}.i = v\}}$$

$$\text{C-Basic}\ \frac{\mathit{Command} \in \{\mathsf{Select}, \mathsf{First}\}}{\Gamma, \mathit{id}_1.\mathit{Command}(\mathit{id}_2 \to \cdots) \Rightarrow \Gamma \cup \{\mathit{id}_1.\mathit{elt} = \mathit{id}_2\}}$$

$$\text{C-Aggregate}\ \frac{}{\Gamma, \mathit{id}_1.\mathsf{Aggregate}((\mathit{id}_2, \mathit{id}_3) \to \cdots) \Rightarrow \Gamma \cup \{\mathit{id}_1.\mathit{elt} = \mathit{id}_3\}}$$

$$\text{C-Zip}\ \frac{}{\Gamma, \mathit{id}_1.\mathsf{Zip}(\mathit{id}_2, (\mathit{id}_3, \mathit{id}_4) \to \cdots) \Rightarrow \Gamma \cup \{\mathit{id}_1.\mathit{elt} = \mathit{id}_3 \wedge \mathit{id}_2.\mathit{elt} = \mathit{id}_4\}}$$

$$\text{C-Assign}\ \frac{V = \mathit{con}(\mathit{expr})}{\Gamma, \mathit{id} = \mathit{expr} \Rightarrow \Gamma \cup \{\bigwedge_{v \in V} \mathit{id} = v\}}$$

Figure 16: List size constraint generation rules. Γ is a set of constraints.

ated is passed to the Z3 SMT solver for resolution. If the constraints are satisfiable, Z3 will produce a *model*, which associates constraint variables to integers that satisfy the constraints. This model is used to derive the needed mapping between identifiers and list sizes.

Type Generation and Function Isolation: Pinocchio requires static sizes on all arrays and loop bounds. To accomplish this, ZØ creates a new struct type for each list with a distinct base type and size in the original program. Each new type has two fields: a static array and a constant defining the size.

Once types for each identifier are established, each sub-expression in the source statement is converted to a function body. To see the need for this step, consider the statement $x.\mathsf{Select}(el \to el.\mathsf{Select}(\ldots))$. C has no expression form for the functionality needed by the Select command, so both expressions must be converted into loop statements. Rather than placing the loop statements in the same method body and carefully managing side effects and sequencing with other sub-expressions, we isolate the emitted code for the inner Select in a separate function, and emit a call to the new function in its place in the context of the outer Select expression.

The statements generated for each LINQ command are straightforward translations of their defined behavior into basic C; in general, the input loop is iterated over, and the lambda passed to the command is invoked over each element. Field lookups, new object construction, and function calls are rewritten to their C equivalents.

Pinocchio Example: Consider the example starting on line 33 of Figure 1. This command traverses the "discount" automata using a list of past purchases. In order to compile this to Pinocchio, we must first solve a set of constraints generated by a traversal of its syntax, as depicted in Figure 16. These constraints relevant to this command are given as:

1. $\mathit{history} \le \mathit{NPurchases}$
2. $\mathit{items.elt} \le \mathit{NItems}$
3. $\mathit{automata} \le \mathit{NEdges}$
4. $\mathit{transducer} \le \mathit{NStates}$
5. $\mathit{purch\_state} \le \mathit{history.elt.3}$

Suppose that we instantiate

$$\mathit{NPurchases} = 500$$

$$NItems = 10,000$$

$$NEdges = 100$$

$$NStates = 50$$

The solver library solves these constraints in less than one second, and produces a model that allows us to resolve static sizes for all of the lists we need to perform the LINQ commands: Select, Aggregate, and First:

$$\{history = 500, items = 10000,$$
$$automata = 100, transducer = 50, purch\_state = 1\}$$

Using these sizes, ZØ generates the necessary input types to emit the LINQ commands. For the sake of brevity, we show only one such type used to represent the variable *automata*, as they all share a similar form:

```
struct Triple100 { Triple Enumerable[100]; }
```

ZØ then begins emitting new functions for each sub-expression in the source statement. For clarity, we will omit some function bodies by in-lining simple sub-expressions into certain function bodies; in reality, ZØ would emit a new function for *every* sub-expression. We begin with the lambda expression passed to the First command:

```
#define Boolean int
Boolean firstPredicate(Triple row, Int32 state,
    Int32 purch) {
  return row._1 == state && row._2 == purch;
}
```

This definition is used to construct the First command in a separate function: The definition First is used

```
Pair First(Triple100 automata, Int32 state,
    Int32 purch) {
  int it;
  for(it = 0; it < Triple100Length; i++) {
    if(firstPredicate(
        automata.Enumerable[it], state, purch))
      return automata.Enumerable[it];
  }
  // Semantics is undefined when
  // Find cannot find the right element
  return automata.Enumerable[0];
}
```

to generate the function for the Aggregate command

```
Int32 Aggregate(Int32500 history,
    Triple100  automata) {
 int it, int0 = 0;
 for(it = 0; it < Pair10000Length; it++) {
  int0 = First(automata, int0, history.Enumerable[it]);
 }
 return int0;
}
```

that traverses the automata: The translation from LINQ to C for this command is straightforward: to aggregate a list, create an accumulator (`int0` in this case), and fold the aggregator function over each element in the accumulator in a loop that covers the entire list. Notice that in the actual code emitted by ZØ, this definition requires a separate function for each new object that is constructed; here, we in-line these functions into `Aggregate` to keep this discussion relatively brief. The entire query can be invoked by calling the resulting Pinocchio program with the relevant inputs to the original program, `history` and `automata`.

5.3.2 ZQL

Recall that we only attempt to convert LinqStmt statements into zero-knowledge, so there are four primary functions to convert, in addition to a few additional expression forms. By no coincidence, the four primary LINQ functions correspond closely to the operations supported by ZQL. Figure 17 gives a set of rewrite rules that can be used to translate a LinqExpr to ZQL's expression language. Select, Aggregate, Zip, and First calls are translated to map, fold, map2, and find expressions. Lambda definitions and functions calls are translated compositionally, by first translating sub-expressions and then building a new construct in the target language. Object creation using new is translated into tuple construction. Recall that user-defined types in a ZØ program must expose a single constructor that assigns all fields of the type; field names are translated into a tuple order using the constructor signature. Similarly, field accesses using fld are translated into a let binding that returns the appropriate tuple component; the translation consults the target identifier's type constructor to deduce the number of fields in the type.

$$\text{T-Select}\ \frac{\mathit{lambda} \Rightarrow \mathit{lambda}_{\text{zql}}}{\mathit{Id}.\mathsf{Select}(\mathit{lambda}) \Rightarrow (\mathsf{map}\ (\mathit{lambda}_{\text{zql}})\ \mathit{Id})}$$

$$\text{T-Aggregate}\ \frac{\mathit{lambda} \Rightarrow \mathit{lambda}_{\text{zql}} \quad \mathit{expr} \Rightarrow \mathit{expr}_{\text{zql}}}{\mathit{Id}.\mathsf{Aggregate}(\mathit{expr},\ \mathit{lambda}) \Rightarrow (\mathsf{fold}\ (\mathit{lambda}_{\text{zql}})\ \mathit{expr}_{\text{zql}}\ \mathit{Id})}$$

$$\text{T-Zip}\ \frac{\mathit{lambda} \Rightarrow \mathit{lambda}_{\text{zql}}}{\mathit{Id}_1.\mathsf{Zip}(\mathit{Id}_2,\ \mathit{lambda}) \Rightarrow (\mathsf{map2}\ (\mathit{lambda}_{\text{zql}})\ \mathit{Id}_1\ \mathit{Id}_2)} \qquad \text{T-First}\ \frac{\mathit{lambda} \Rightarrow \mathit{lambda}_{\text{zql}}}{\mathit{Id}.\mathsf{First}(\mathit{lambda}) \Rightarrow (\mathsf{findt}\ (\mathit{lambda}_{\text{zql}})\ \mathit{Id})}$$

$$\text{T-Lambda}\ \frac{\mathit{expr} \Rightarrow \mathit{expr}_{\text{zql}}}{(\mathit{Id}_1, \mathit{Id}_2, \dots)\ \rightarrow \mathit{expr} \Rightarrow \mathsf{fun}\ (\mathit{Id}_1, \mathit{Id}_2, \dots)\ \rightarrow \mathit{expr}_{\text{zql}}}$$

$$\text{T-Call}\ \frac{\mathit{expr}_i \Rightarrow \mathit{expr}^i_{\text{zql}}}{\mathit{Id}(\mathit{expr}_1, \dots, \mathit{expr}_n) \Rightarrow \mathit{Id}(\mathit{expr}^1_{\text{zql}}, \dots, \mathit{expr}^n_{\text{zql}})}$$

$$\text{T-Fld}\ \frac{\mathsf{typeof}(\mathit{Id})\ \text{has}\ k\ \text{fields}}{\mathit{Id}.\mathsf{fld}(n) \Rightarrow (\mathsf{let}(\mathit{Id}_1, \dots, \mathit{Id}_n, \dots, \mathit{Id}_k) = \mathit{Id}\ \mathsf{in}\ \mathit{Id}_n)}$$

$$\text{T-NewNamed}\ \frac{\mathit{expr}_i \Rightarrow \mathit{expr}^i_{\text{zql}}}{\mathsf{new}\ \mathit{Id}(\mathit{expr}_1, \dots, \mathit{expr}_n) \Rightarrow (\mathit{expr}^1_{\text{zql}}, \dots, \mathit{expr}^n_{\text{zql}})}$$

$$\text{T-NewAnon}\ \frac{\mathit{expr}_i \Rightarrow \mathit{expr}^i_{\text{zql}}}{\mathsf{new}\ \{\mathit{Id}_1 = \mathit{expr}_1, \dots, \mathit{Id}_1 = \mathit{expr}_n\} \Rightarrow (\mathit{expr}^1_{\text{zql}}, \dots, \mathit{expr}^n_{\text{zql}})}$$

Figure 17: Transformation from ZØ LINQ to ZQL expressions.

Example: To illustrate the process of converting a ZØ LINQ statement to ZQL with rewrite rules, consider the example given in Figure 1. As previously discussed, the statement beginning on line 33 traverses an automata using the user's shopping history to arrive at a discount. Applying the rules from Figure 15, we start with T-Aggregate. The precondition of this rule states that both the initial accumulator and the lambda portions of our LINQ command must have valid ZQL translations. The initial accumulator is the constant 0, which is already valid ZQL.

Moving on to the lambda subexpression, we need to derive a translation for the expression body, which is another LINQ expression that performs a search using `First` over a list of triples. Descending recursively, we see that to translate the `First` command, we need to find a valid ZQL translation for the find predicate passed to the command. This is mostly straightforward, but requires an application of T-Fld to de-compose the `Triple` comprising each list element into its constituent `Int32` values. The only precondition of this translation is that the type of `Triple` has k fields for some k; this is true for $k = 3$. So, we can rewrite:

$$\texttt{trans.fld}\langle\texttt{int}\rangle((1)) \Rightarrow (\texttt{let}\ (\_1, \_2, \_3)\ =\ \texttt{trans in}\ \_1)$$

We can do the same for $\texttt{trans.fld}\langle\texttt{int}\rangle(2)$. The field accesses are used in a conjunctive equality test, which is translated compositionally using T-Op. With these rewrites, the final result for the find predicate is:

```
fun(trans) →
      (let(_1, _, _) = transin(_1 = state))
      @&(let(_, _2, _) = transin(_2 = purch))
```

Plugging this expression back into T-Aggregate, we arrive at the following for our final rewrite:

```
                              fold
(fun(state, purch) →
  find(fun(trans) →
  (let(_1, _, _) = transin(_1 = state))
  @&(let(_, _2, _) = transin(_2 = purch))
automata
```

This functionality is invoked on the input `history`; the expression is incorporated into an outer `query` function, which is called on LINQ to ZQL translations of each of the region's inputs.

	Description	Trusted hardware	Distributed computation	Streaming computation	Multiple ZQL stages	Parallelizable computation	New primitives
FitBit	6.1	✓	✗	✗	✗	✓	✗
Slice	6.6	✗	✓	✗	✓	✓	✓
Traffic	6.4	✓	✓	✓	✓	✗	✓
Loyalty	6.3	✗	✗	✗	✗	?	✓
CNIDS	6.5	✓	✗	✓	✗	?	✗
Studies	6.2	✓	✗	✓	✗	✓	✓

Figure 18: Case studies: a classification and a guide.

6 Motivating Case Studies

This section presents 6 case studies. The companion technical report presents architectural diagrams and a detailed description of the algorithm for each application. Below we present a taxonomy of our applications along six dimensions that relate to practical deployment concerns.

Trusted hardware: Does this application require trusted hardware to establish integrity for the inputs? The three applications that do not have this requirement make a different trust assumption: the source of the data is trusted by the verifier, but not able to violate their privacy concerns.

Distributed computation: Does this application require some kind of peer-to-peer distributed computation? Both Slice and the Traffic Density application require multiple provers to share intermediate data using peer-to-peer communications.

Streaming Computation: Does this application require the ZQL portion of the implementation to continuously accept and process input data, providing the verifier with a continuous stream of results and proofs?

Multiple ZQL stages: Does this application require rounds of iterated ZQL computations, interleaved with intermediate processing outside of ZQL? For example, both Slice and Traffic require an initial round of secret share generation (in ZQL), followed by a peer-to-peer transmission of shares (not in ZQL), followed by a round of share aggregation (in ZQL). This property suggests the need for a unified development and compilation framework, that takes care of the transitions between these stages.

Parallelizable: Is this application inherently parallelizable? Two applications, Loyalty and CNIDS, are marked as "maybe" because their primary functionality relies on some form of automaton evaluation. This type of functionality may be parallelizable if an extended form of lookup table is eventually supported by the ZQL compiler.

New primitives: Does this application require new primitive support from the ZQL compiler? Three applications require either map2, fold2, or both.

Figure 18 shows a classification of our case studies along the dimensions outlined above. In each cell, ✓ corresponds to yes, ✗ to no, and ? to maybe. Similarly to [19], we assume that the sensor readings devices can are trusted and untampered with, and come signed by their producer, but the machine or mobile phone (Client tier) that performs the distance computation is not. Techniques for building trust deeper into the platform are complimentary to our work [28].

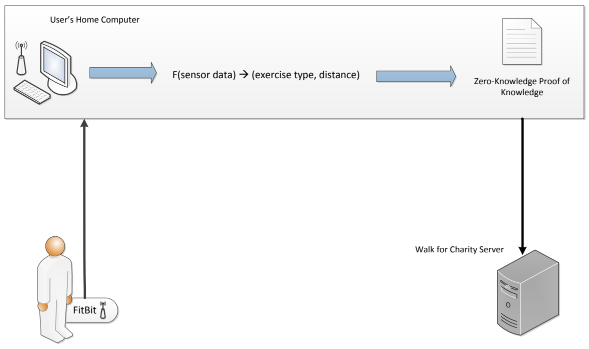
6.1 Walk for Charity with FitBit

Several programs exist for paying users for the amount of physical exercise they perform, either directly in the form of rewards, or indirectly by making charitable donations on their behalf, such as `earndit.com`. This works by requiring users to log their exercise habits using a FitBit or other sensor device (e.g., a GPS-enabled tracker)to measure the distance the user walks, runs, or bikes, and send the logs to a centralized server.

Privacy Concern: The user may not want to reveal their *exercise route* to a relatively untrusted third party.

Integrity Concern: The service is spending money on the basis of distance derived from sensor logs. If the distance computation can be subverted, the possibility for fraud arises, analogously to pay as you drive insurance [5, 42, 45].

Proposed Solution: Keep all sensor readings local to the user's machine (laptop or mobile device), perform the distance computation locally, on the client, send the result of the distance computation to the centralized third-party server. Use ZKPK to ensure that the distance computation is performed correctly. This approach is similar to what has been advocated for smart metering [39].


User's Home Computer
F(sensor data) → (exercise type, distance)
Zero-Knowledge Proof of Knowledge
Walk for Charity Server
FitBit

The algorithm proceeds as follows:

1. After each GPS reading, the user's fitness device sends an encrypted commitment to the Walk for Charity server, as well as the user's home computer.
2. At the end of the day (or during some other downtime), ZQL code running on the user's computer processes the GPS readings, and computes the total distance walked by the user.
3. The results of the ZQL computation, as well as their correspond proof, are sent to the Walk for Charity server for validation.

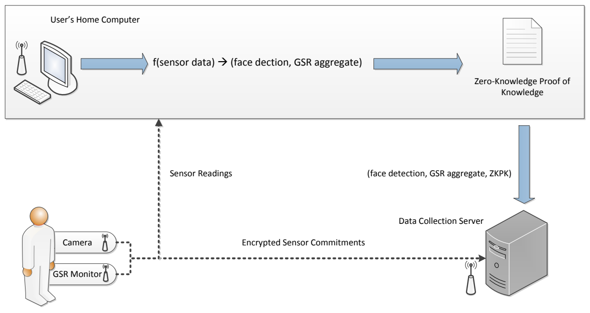
6.2 Supervised Studies in Social Sciences

Many scientific studies, especially in medical and social sciences, require subjects to wear sensors and undergo protocols that provide information about their physiological and psychological state. A study that seeks to understand the effect of common workplace events on worker's stress levels might require a participant to wear a galvanic skin response sensor and a camera to detect face-to-face interactions.

Privacy Concern: Participants may have concerns about the use of their physiological measurements or, most prominently, the processing of images taken from their cameras.

Integrity Concern: These studies typically involve payment given to subjects. Subjects concerned about their privacy, or those who simply do not want to wear intrusive sensor devices, have an incentive to *fake* the data used in the study.

Proposed Solution: Have all sensors associated with the study report readings to the subject's machine (desktop or mobile phone). This machine performs aggregate computations relevant to the actual study on the readings, reporting results and discarding the raw sensor readings. ZKPK is used to ensure that the readings are processed correctly. As above, this assumes that the sensors are trusted and untampered with, but that the subject's machine is not.


User's Home Computer
f(sensor data) → (face dection, GSR aggregate)
Zero-Knowledge Proof of Knowledge
Sensor Readings
(face detection, GSR aggregate, ZKPK)
Data Collection Server
Encrypted Sensor Commitments
Camera
GSR Monitor

The algorithm proceeds as follows:

1. At each time interval t, the sensors attached to the subject's body take a reading, and send it to the subject's workstation, as well as sending an encrypted commitment to the data collection server.
2. The subject's workstation performs aggregate computations over the readings, and sends the

results, along with a zero-knowledge proof of correctness, to the data collection server.

- The images are processed using a face detection algorithm based on Principal Components Analysis. A set of "eigenfaces" and their corresponding eigenvalues are assumed public input to the algorithm, and come pre-trained from outside data. The algorithm simply projects the image from the camera into "face space", and computes its distance from the average face computed from the training set. The distance is returned from the computation.
- At the moment, the GSR readings are fed through an identity map, as the primary threat to privacy in this scenario is currently presumed to be the images taken from the subject's camera. However, this aspect of the algorithm could be changed to return the mean or mode of some public number of samples.

3. The data collection server collects the proofs, and verifies them against the encrypted commitments sent by the sensors.

6.3 Personalized Loyalty Cards

Many of today's large retailers such as Target, Best-Buy, etc. use customer loyalty cards to encourage repeat visits. Typically, the customer must enroll in a loyalty program, and receive a card that can be applied to receive discounts in future visits. Recently, certain retailers (e.g., Safeway) have begun personalizing this process by using the customer's past purchase history (available because of the association between checkout and loyalty card) to create discounts available only to one particular customer. Depending on the retailer, these discounts can be sent to the customer's mobile phone, or applied automatically at checkout.

Privacy Concern: Many people are not comfortable with a retailer tracking their purchases. This is most readily illustrated by a recent scandal with Target discovering that a teenage girl was pregnant before her parents did [16].

Integrity Concern: Retailers offer discounts on the basis of past purchase history. If a customer were able to fake a purchase history, they might be able to obtain a discount for an item of their choosing. Moreover, having a reproducible strategy for "generating" discounts might create a serious problem for the retailer, similar to those experienced by some retailers that were overly generous in offering Groupons[2].

Proposed Solution: The "loyalty app" on the customer's phone takes the place of the traditional card. At checkout, the app uses a near-field communication sensor with the register to receive a list of purchased items. This information is stored locally, and never sent to the store's servers. Personalization algorithms are applied to this sensor data on the phone, and the results are discounts that can be used at the next transaction. These discounts are transmitted to the register at the time of purchase, and ZKPK is used to demonstrate that the correct algorithms were applied to the NFC sensor data received in previous transactions. This assumes that the sensor readings are trusted, but the mobile app is not.

The algorithm proceeds as follows:

1. Before checkout time (possibly during the phone's downtime, or on the user's workstation), a ZQL query is executed to associate the user's previously certified transactions (see Step 2) with a set of discounts offered by the store. This is performed by processing the user's transaction history with a finite-state transducer. This assumes that the user gives his transaction history to the ZQL query in a particular order, which is checked by the query. The choice of using a finite-state transducer to associate discounts with transaction histories makes this model general with respect to the store's system of discounting. Decision trees and arbitrary sets of rules can also be encoded using this construct.
2. At checkout time, the user waves his smartphone at the register. The register certifies the GUID of

[2]`http://risnews.edgl.com/retail-trends/Is-Groupon-a-Raw-Deal-for-Retailers-73442`

each item purchased, and sends encrypted commitments to the smartphone via NFC, as well as recording them in a central database. The user's phone does not transmit any identifying information to the register, so the store is unable to associate the purchases with the user.

3. After receiving the certified transaction list for the current purchases, the user's smartphone sends the result of the transducer from Step 1, and its corresponding proof. The register can then provide the discounts.

6.4 Crowd-sourced Traffic Statistics

Several mobile applications such as Waze (`waze.com`) and Google Maps provide traffic congestion information to end-users based on the combined GPS readings of everyone using the app.

Privacy Concern: Users do not want to share their location with the app's servers, or the general public (in the case of a distributed protocol).

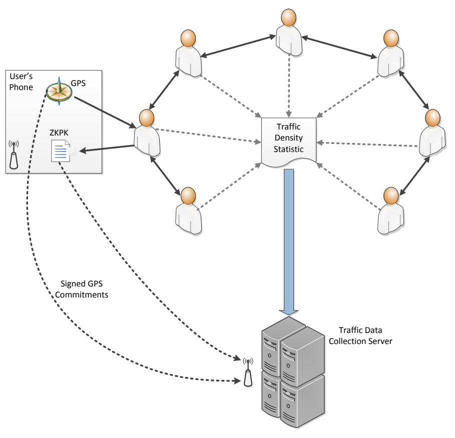
Integrity Concern: The app needs reliable GPS readings from users to provide its core functionality. If users wish to "game" the system by providing fake GPS readings while receiving the end-product, the integrity of traffic data is compromised for everyone.

Proposed Solution: Let the users keep their GPS readings local, and take part in a distributed protocol to compute local density information for transmission to the app's central server. Clients represent their location on a map using a vector, represented as a set of secret shares, which can be added to the other clients' vector shares to derive the overall traffic density map. When each client sends their summed shares to the server, it can reconstruct the density map by combining the shares.

In more detail, this algorithm works as follows:

1. At regular intervals, the collection server sends a request to each client for traffic density statistics. Density statistics are represented by partitioning the map into regions, and counting the number of clients in each region.

2. On receiving a request, each client:



   - Takes a GPS reading, and has it signed using a trusted subsystem in the operating system. An encrypted commitment of this reading is forwarded to the collection server.
   - The client computes its region, and executes a ZQL query to: (1) check that its computed region number is correct; (2) compute a set of linear secret shares of its region number. Assuming R regions, the region numbers used by the algorithm are:

$$R^0, R^1, \ldots, R^(R-1)$$

   - The client sends its secret shares to all other clients, along with the proof that each share was computed correctly.
   - On receiving the other clients' secret shares, the client adds them all together, and forwards the result, along with the proof that the shares were added correctly, to the collection server.

3. On receiving all shares, the collection server interpolates to learn the sum computed by the clients. The server than then compute the number of clients in each region by converting it into its base-R representation.

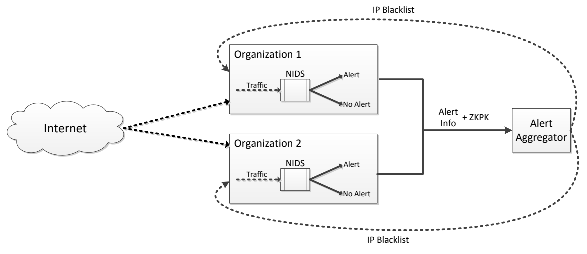
6.5 Collaborative Network Intrusion Detection

Collaborative intrusion detection (CNIDS) has long been a goal of security practitioners [30]. In the CNIDS scenario, multiple (distrustful) organizations share the results of their network intrusion detection sensors, to provide their peers with advanced warning about possible threats. A practical approach involves sharing IP blacklists: when an IP generates a valid NIDS alert on one organization's network, the IP is recorded and sent to the other participating organizations.

Privacy Concern: NIDS operate on highly sensitive data — raw network traces. Organizations participating in CNIDS do not want to share their traces with other organizations, and in many cases, may be prohibited from doing so by law or organizational policy.

Integrity Concern: Given the privacy concern and the benefits of participating, some organizations may want to freeload by suppressing their own NIDS alerts. Additionally, if an adversary manages to compromise a participating network, it may choose to suppress or even generate false alerts, which may result in a denial of service for the targeted IP address.

Proposed Solution: Provide a ZKPK for the NIDS signature-matching process, to prove that a claimed intrusion is correct according to the signature. Note that this approach assumes that raw network data coming into the NIDS has not been tampered with, but that the machine performing the signature matching may not be trusted.



The algorithm works as follows:

1. As network events come in, the NIDS machine certifies them, and sends encrypted commitments to a separate machine; for our purposes, call this machine the prover.

2. After a pre-defined number of network events have transpired, or an alert has been raised, the prover runs a ZQL query to traverse a finite state machine representing the NIDS signature using the certified network events. The query returns the IP address of the network event that caused the alert (if an alert resulted), or 0 otherwise. The prover sends the result of the traversal (the , as well as its ZK proof and the encrypted network traffic commitments, to the alert aggregator.

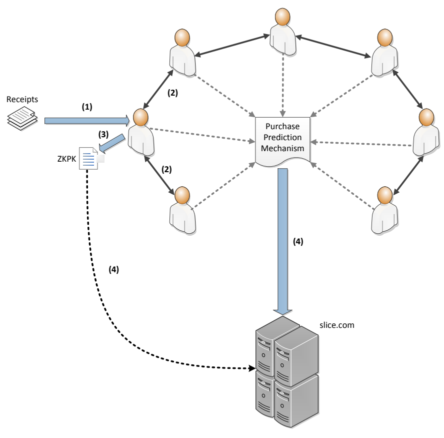
6.6 Slice: Organizing Shopping

Slice (`slice.com`) is a service that takes as input a user's past purchase history from their email mailbox, and provides various services using that data. One such service is product recommendation — given everybody's past purchase history, slice can build classifiers that predict a likely "next" purchase.

Privacy Concern: Handing one's entire purchase history to a profit-driven third party has obvious privacy implications. So does the troubling need to share one's email credentials with Slice at the moment.

Integrity Concern: A user, particularly one concerned about privacy, might provide fake data to Slice in order to obtain the useful classifier, which would pollute Slice's data for everyone and jeapordize Slice's ability to profit from the classifier.

Proposed Solution: Keep the user's purchase history local, and have the users take part in a distributed protocol in order to produce the classifier for Slice. Use ZKPK to ensure that no user is able to subvert the distributed classifier computation. This approach assumes that the purchase history data used by the distributed learning algorithm is trusted. Although it is not immediately clear how this end might be accomplished, one solution might be to have the retailer certify each purchase, and send a commitment along with the user's receipt. This would achieve a similar level of trust to the current Slice implementation.



The approach produces a random forest classifier from the collective purchase history of all Slice users, with the goal of predicting whether a particular user is likely to purchase a given item. Assuming that we have a single, centralized database of purchase histories stored using the schema given above, a random forest would be constructed by the following algorithm (inspired by the relevant Wikipedia article):

For each tree, perform the following randomized computation:

1. Let m be the number of input variables used to determine the decision at each node of the tree.
2. Choose a random subset of n rows of the database to be used for training the current tree.
3. For each node of the tree, randomly choose m variables on which to split. Calculate the best split based on the
4. value these variables take in the training set.
5. Grow the tree to a specified depth, and do not prune it.

The classifier is built by combining all trees. Classification is performed by traversing each tree for the given sample, and taking the statistical mode of the label associated with each traversed leaf node.

Our setting is slightly different: rather than having a centralized dataset, each row is housed on a different user's device. The users do not wish to share their row with Slice, so this algorithm must be run in a distributed fashion by sending queries to each user corresponding to the rows selected in step 1.2 of the algorithm given above. On receiving a query, the user invokes ZQL functionality (given below) to compute the correct answer based on their purchase history, and sends the query result and its zero-knowledge proof of correctness back to Slice. This is given in the following algorithm:

Our setting is slightly different: rather than having a centralized dataset, each row is housed on a different user's device. The users do not wish to share their row with Slice, so this algorithm must be run in a distributed fashion by sending queries to each user corresponding to the rows selected in step 1.2 of the algorithm given above. On receiving a query, the user invokes ZQL functionality (given below) to compute the correct answer based on their purchase history, and sends the query result and its zero-knowledge proof of correctness back to Slice. This is given in the following algorithm. For each tree, perform the following randomized computation:

1. Let m be the number of input variables used to determine the decision at each node of the tree.
2. Choose a random subset of n users to train the current tree. Label them $u_1, \ldots, u_n$.
3. For each node of the tree, randomly choose m variables on which to split. Label them $v_1, \ldots, v_m$.
   - Send a query q pertaining to each of the m variables to $u_1, \ldots, u_n$. Construct q by:
     $$q = l_1 \leq c_1 \leq u_1 \wedge l_m \leq c_m \leq u_m \wedge \ldots$$
     where c_j correspond to the amount spent in category j, and item i is the class label that we are trying to deduce.
   - Each user constructs a set of linear secret shares of his query result, and sends them

to the other $u_1, \ldots, u_n$ (excluding himself), along with the proof that the share is constructed appropriately from the user's inputs.

- The users add their shares independently, and send the result (and its ZK proof) to Slice.
- Slice interpolates on the shares returned by the users, to obtain the number of users that match the query q.

4. Based on the query results given by $u_1, \ldots, u_n$, calculate the best split, and construct the new node.

5. Grow the tree to a specified depth, and do not prune it.

The classifier is built by combining all trees. Classification is performed by traversing each tree for the given sample, and taking the statistical mode of the label associated with each traversed leaf node.

Scaling	ZØ scales to all application configurations. Others may time out on small settings: 100-byte traces (NIDS), >100 peers (Slice).
Latency	ZØ improves up to 58×, ≈ 15× on average
Proof size	ZØ almost always less than 1 MB, at most 1.5 MB. ZQL proofs can be tens or hundreds of MBs.
Global tradeoffs	ZØ may be slower at one tier (4× slower for Slice server), but savings at other tiers is always much greater (16× faster for Slice clients)

Figure 20: Performance summary.

	Pin. Speedup		**ZQL Speedup**		**Avg. Speedup**	
	Avg.	Max	Avg.	Max	Avg.	Max
FitBit	31.7	**31.7**	5.2	5.26	18.5	18.5
Study	1.0	1.0	13.9	14.6	7.5	7.8
Loyalty	9.4	16.7	5.4	8.2	7.4	12.5
Waze	9.4	23.1	31.1	**43.8**	20.3	**33.4**
CNIDS	34.2	**34.2**	2.1	2.3	18.2	18.25
Slice	6.0	16.4	19.8	**58.1**	12.9	**37.3**
Average	**15.3**		**12.91**		–	

Figure 21: Latency speedup factors for each application.

7 Experimental Evaluation

All experiments were performed on a Windows Server 2008 R2 machine with two 3.6 GHz 64-bit cores. All reported timing measurements correspond only to the zero-knowledge portion of the application's execution time, as this is the only portion that our compiler attempts to optimize.

Note that we used early pre-release versions of both ZQL and Pinocchio compilers, obtained from the respective authors between December 2012 and March 2013. We anticipate that both tools will improve in terms of performance and perhaps proof size and memory efficiency with the passage of time. This can come about in part of better parallelism in ZQL and more efficient equality comparisons in Pinocchio.

The execution time of the ZK code is generally much higher that of the rest of the application, so focusing on these parts gives an accurate picture of the overall execution time. Each zero-knowledge proof generation and verification task was terminated after ten minutes. Our implementation uses 1,024-bit RSA keys for ZQL computations. Integers in Pinocchio circuits were configured to have 32-bits for comparison operations, and operate over a 245-bit field.

Figure 20 summarizes the key performance results from our experiments. We found that the ZØ-generated code gave significant performance benefits both in terms of computation time and proof size: up to 58× runtime speedup, with most proofs below 1 MB (the largest being ≈ 1.5 MB). Furthermore, we saw that global optimization is necessary to arrive at an ideal performance profile: some applications perform noticeably worse at one tier, but in each case the speedup at another tier was always greater. For example, the code ZØ generated for the Slice server ran ≈ 4× slower than Pinocchio's, but the client tier experienced ≈ 16× reduced latency.

Figure 21 shows the latency speedups across all applications. The average speedup delivered by ZØ is 15.3× compared to Pinocchio and 12.91× compared to ZQL.

In slightly more detail:

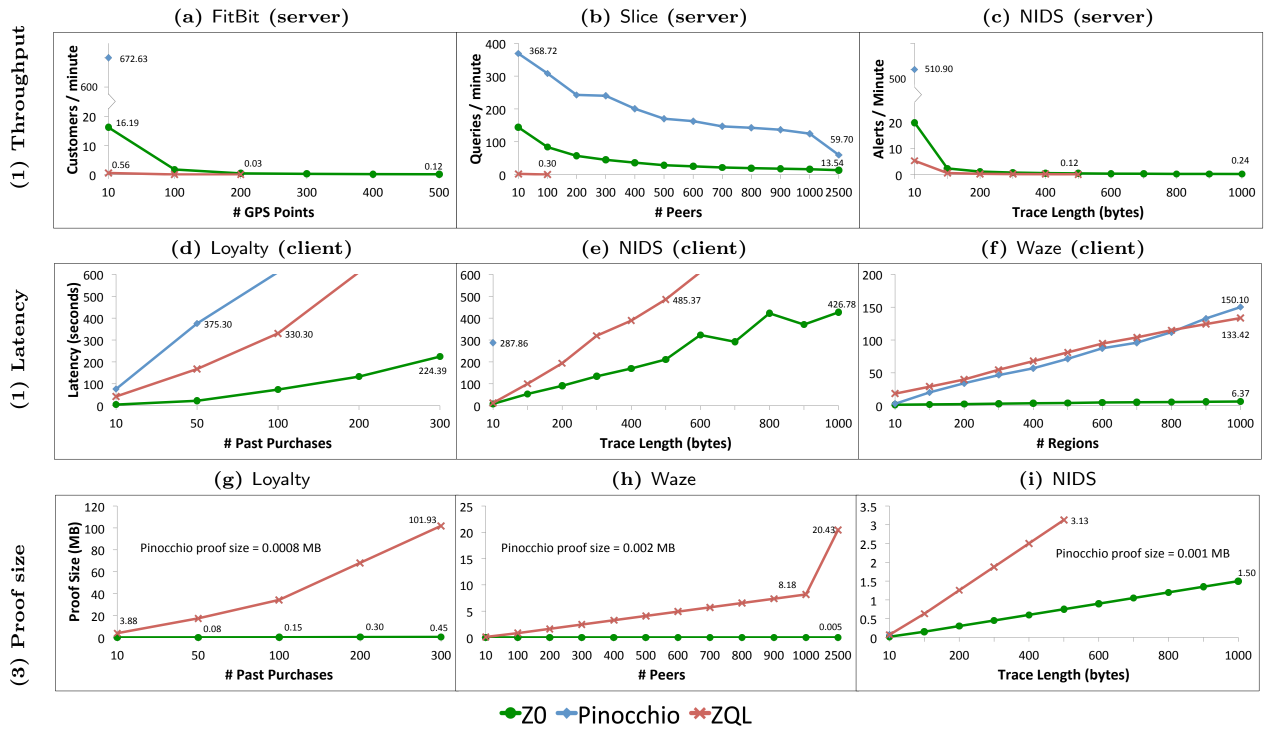


Figure 19: (1) Throughput, (2) latency, and (3) proof size for a characteristic sample of application functionality.

- The ZØ solution is the only one that scales to all settings used in our experiments. Often, ZQL or Pinocchio are only able to complete the smallest configuration before timing out.
- The ZØ solution also gives significant savings on latency: as much as 44× on some applications, and commonly around 5×–10×.
- Our experiments highlight the need for our global optimization algorithm (Section 4.5): in several experiments, the ZØ at one tier performed more slowly than an alternative (about 4× less throughput in the case of Slice's server code), but in each case, the savings provided at a different tier was much greater (about 16× less latency for Slice's client code).
- The ZØ solution nearly always produces small zero-knowledge proofs, typically less than one megabyte, and at most 1.5 megabytes in our experiments. Contrasted with the size of ZQL proofs, which can be tens or hundreds of megabytes in many cases, this amounts to a significant savings.

Results: Space limitations do not allow us to present our measurements exhaustively. Instead, Figure 19 shows a sample of the runtime characteristics for our target applications. Rather than giving raw execution times, the results are broken into three categories: *throughput*, *latency*, and *proof size*. These metrics were selected to more clearly depict the impact of zero-knowledge techniques on each application.

Throughput: Figure 19(a)–(c) shows the results of three experiments involving throughput. Figure 19(a) shows the server's throughput for the FitBit application, which corresponds to the number of customers per minute the server can handle as the size of each customer's workout (n) increases. First, notice that the only point at which the Pinocchio solution is present is $n = 10$, where it far outperforms the other two solutions. This is because verification in Pinocchio is very fast, whereas the time to construct a proof can be quite slow: in this case, for any $n > 10$, the proof construction phase timed out after ten minutes. Similarly, for $n > 200$, the ZQL solution times out. With the exception of $n = 10$, the ZØ solution dominates the others, offering 35× improvement at $n = 100$ and 14× at $n = 200$, while remaining the only viable option for higher values of n.

Figure 19(b) shows the number of random forest construction queries per minute the Slice server is able to handle, as the number of participating peers increases. This result is particularly interesting, as it motivates the need to perform global optimization over both the client and servers' computations to achieve reasonable performance. In this figure, the Pinocchio solution dominates the ZØ solution at all data points. This seems like a negative result, until one considers the amount of time needed at the client to answer a query, where the Pinocchio solution is up to 16× slower than the ZØ solution. This validates the compiler's choice of back-ends, as the penalty on the client is much greater than that on the server.

Figure 19(c) shows the number of intrusion alerts per minute the collaborative NIDS server can handle as the number of bytes in the intrusion trace increases. Again, as with Figure 19(a), Pinocchio outperforms at a single extremal small data point, but fails to scale to any larger points. For the remaining points, the ZØ solution outperforms the others by about 4×, and is the only solution that is able to scale to even the modest intrusion trace length of 1 KB.

Latency: Figure 19(d)–(f) shows the results of three experiments involving latency. Latency is always measured in seconds, and has a uniform upper bound of 600 seconds, which corresponds to our experimental timeout. Figure 19(d) shows the latency of the client side of the Loyalty application as the number of purchases used to personalize discounts (n) increases. The ZØ solution far outpaces both alternatives at all data points (5×–17× improvement), and is the only solution that scales past $n = 100$. For longer purchase histories, the ZØ solution completes in around 2.5 minutes, which is ample time if the application is location-aware and begins proving a set of discounts when the user enters the store.

Figure 19(e) shows the NIDS client's latency to demonstrate that a single intrusion is present in a trace. Again, the ZØ solution is the only one to scale to a modest trace length of 1K, while the Pinocchio times out at all points beyond ten bytes. Otherwise, we see that as long as intrusions are spaced more than seven minutes (426 seconds) apart, the NIDS client has enough time to build proofs for each intrusion trace.

Figure 19(f) shows the latency of the Waze client to send traffic statistics for a single location query as the size of the map (n) increases. First notice that the ZØ solution is essentially constant, not varying by more than 3.5 seconds between any two data points; this is the only feasible solution, as both alternatives may require up to two minutes to finish computation, which will limit the quality (i.e., recency) of the statistics the server is able to gather over time. Second, notice that at about $n = 800$, ZQL becomes more performant than Pinocchio. This is because as the map increases, the size of the lookup table needed to encode the regions increases. Pinocchio is not able to perform lookups as quickly as ZQL, so the portion of the computation needed for lookups becomes more significant at higher values of n. ZQL performs worse at lower values because most of the computation corresponds to the multiplications needed to compute secret shares, which it does not complete as quickly as Pinocchio.

Proof Size: Figure 19(g)–(i) shows the results of experiments involving the size of the zero-knowledge proof in various applications. We always measure in megabytes, and do not display a curve for the Pinocchio solutions, as it is constant across input size and is usually too small to distinguish on the same scale as the ZQL and ZØ solutions. Figure 19(g) shows the proof size for the Loyalty application as the number of past purchases (n) varies. While the Pinocchio solution of course dominates the others by this metric (864 bytes), as we know from previous experiments (Figure 19(d)) it does not scale in terms of Latency. The ZØ proof size remains nearly constant, always under 500 KB, whereas the ZQL solution requires at least three megabytes (to perform the inequality checks at the beginning), and finishes at about 100 megabytes. Note that we obtained the point at $n = 300$ despite the timeout, by letting the prover run for longer in this single instance. Because the Loyalty application needs to communicate this proof wirelessly to a POS terminal, size is crucial, and the ZØ solution offers the best overall characteristics in terms of size and latency.

Figure 19(h) shows the proof size for the Waze application as the number of peers varies. Again, Pinocchio dominates (2 KB), but the tradeoff in latency for this proof size is quite high (Figure 19(f)). The ZØ proof size remains constant at around 5 KB because the only processing done by ZQL is table lookups, which have a constant proof size. The ZQL solution requires 20 megabytes for 2,500 clients, and 8 megabytes for 1,000 clients, making it untenable given that the clients need to transmit proofs frequently over cellular networks.

Figure 19(i) shows the proof size for the NIDS application as the intrusion trace length increases. The Pinocchio proof is about 1 KB, but again the tradeoff in latency makes this characteristic mostly irrelevant. The sizes for the ZØ and ZQL solutions are both linear, with the ZØ solution offering a savings of about 4× at all data points. This is a significant savings, considering that false positives may be frequent, so the client may need to send proofs to the server almost continuously throughout service.

8 Related Work

Tier-Splitting and Language Methods: A number of compilers exist that enable automated tier-splitting in some form. In the context of web programming, Volta [29], Links [13], and Hilda [47] were among the pioneering efforts. More recently, the Google Web Toolkit [24] has gained popularity as a monolithic, tier-splitting framework for web application development. It shares a few similarities with Volta, in that developers supply their application as a single piece of code in a high-level language, such as Java or C#. They specify which tier each method will execute on, either the client or server, and the compiler generates byte code for the server and JavaScript for the client, inserting data transfer and synchronization code automatically. ZØ draws inspiration from this work in its tier-splitting functionality: mechanically, there is little difference in how ZØ handles tier splitting from these tools. The difference is in how ZØ "drives" tier-splitting: cost models including execution time and data transfer size are used to derive an optimization problem whose solution represents an ideal division of functionality between tiers.

Others have used tier splitting with a focus on security and privacy guarantees. SWIFT [12] builds on the JIF [33] language, which incorporates *security types* into a Java-like language to provide confidentiality and integrity guarantees. In their setting, integrity takes a slightly different meaning than in ours: it is an information flow property that ensures *trusted* data is not affected by *untrusted* sources. SWIFT builds on JIF by supporting tier-splitting for web applications, providing the guarantee that data that is private to the server (or client) is not sent to the client (or server). To accomplish this, information flow constraints are embodied into an integer programming problem whose solution corresponds to a valid (e.g., secure) placement of code and data onto tiers that minimizes the number of messages communicated between the two. ZØ supersedes the optimization aspect of this work by incorporating computation time and data size into the tier placement algorithm.

Backes *et al.* [3] presented a compiler for distributed authorization policies written in Evidential DKAL [7] (a variant of the Distributed Knowledge Authorization Logic that supports signature-based proofs). The authorization policies may be privacy-aware, so that principals can prove their right to access a resource based on sensitive information, without directly revealing the content of that information. Like ZØ, zero-knowledge proofs are used to support this functionality. ZØ differs from this work in its application focus: whereas ZØ allows developers to specify functionality in a subset of C# and integrate with existing .NET code, the work of Backes *et al.* translates declarative statements in a high-level distributed authorization logic into executable cryptographic code with zero-knowledge properties.

A notion of integrity similar to that used in ZØ has been addressed in tier splitting by Ripley [44]. Ripley prevents client-side cheating in web applications by replicating on the server the code that *should* execute on the client, and checking that the outputs produced by the client match those that are expected by the replicated computation. Unlike ZØ, this mechanism does not preserve privacy.

Protection against untrusted clients has also received much attention in the context of online gaming [25, 46]. In a distributed online game, part of the application workload is typically delegated to the clients and the server keeps track of only an abstract state of the game environment. Jha *et al.* propose a solution to the distributed online game integrity problem by performing random audits of the client state verifying that the client has not manipulated its state in violation of the semantic rules of the game [26]. Our approach, in contrast, provides a non-probabilistic guarantee of integrity at a potentially higher cost, and also preserves privacy.

Zero-Knowledge Proofs: Zero-Knowledge proofs of knowledge [6] have seen extensive use in the privacy and applied cryptography literature. Zero-knowledge protocols have been developed for proving linear relations [8], equality and inequality [40], logical connectives [8], multiplication [9], division and modulo [10], and set membership [9]. More recently, Gennaro *et al.* have explored the use of *quadratic span programs* for providing zero-knowledge proofs of knowledge [22]. Their approach allows very efficient proofs for certain types of computation, including matrix operations and hash computations; additionally, the proof size remains constant regardless of the nature of the computation, or the size of the inputs. Taken together, all of this work on zero-knowledge proofs of knowledge allows one to provide a zero-knowledge proof that expresses the functionality of an arbitrary circuit, as in the case of fully-homomorphic encryption [23].

As a result of this work, several projects have sought to provide general-purpose zero-knowledge compilers [2, 3, 19, 31, 36] that take as input a proof goal, and produce executable (oftentimes distributed) code for a zero-knowledge proof of that goal. The first set of zero-knowledge proof compilers [2, 3, 31] took as input specifications of cryptographic protocols, such as in the Camenisch-Stadler notation [11]. While this allows those responsible for building the zero-knowledge proof-based computations with more control over the specific parameters used in the scheme, it is less suitable for developers without extensive expertise in cryptography because computations cannot be expressed in the familiar terms of a high-level programming language. The second group of zero-knowledge compilers [19, 36] are specifically geared towards generating proofs for general-purpose computations, and allow developers to specify their zero-knowledge computations in a language similar to F# (ZQL, [19]) or a subset of C (Pinocchio, [36]). Our work makes extensive use of this generation of zero-knowledge compilers, as we attempt to match specific computations to the backend most appropriate for handling tme.

There are a number of larger projects that incorporate zero-knowledge proofs in order to manage integrity without sacrificing privacy. Recently, Rial and Danezis proposed a system for privacy-preserving smart metering [39] in which the client uses a zero-knowledge proof of knowledge to demonstrate a correct billing total without releasing the readings made by the meter. Danezis *et al.* described zero-knowledge random forest and hidden Markov model classification protocols [14]. This work is particularly relevant for our third-party classifier model, as it provides algorithms for performing a variety of types of classification in zero-knowledge. Balasch *et al.* proposed a privacy-preserving automotive toll pricing protocol [4], based on zero-knowledge proofs of knowledge similar to those described here.

9 Conclusions

This paper paves a way for using zero-knowledge techniques for day-to-day programming. We have described the design and implementation of ZØ, a distributing zero-knowledge compiler which produces distributed applications that rely on ZKPK to provide simultaneous guarantees for privacy and integrity. We build on recent developments in zero-knowledge cryptographic techniques, exposing to the developer the ability to take advantage of these advances without requiring domain-specific knowledge or learning a new specialized language. Most of the heavy lifting is done by the compiler, including cost modeling to decide which zero-knowledge back-end to use and how to split the application for optimal performance, together with the actual code splitting.

Our cost-fitting models provide an excellent match with the observed performance, with R^2 scores between .97 and .99. Our global application optimizer is fast, completing in under 3 seconds on all programs. Our manual and experimental examination of program splits and back-end choices proposed by ZØ confirms that they are indeed optimal. Using 6 applications based on real-life commercial products, we show how ZØ makes it viable to use zero-knowledge technology. We observe performance improvements of over 58×. Perhaps most importantly, ZØ allowed many of the applications to scale to large data sizes with thousands of users while remaining practical in terms of computation time and data size. This means that applications which were not feasible using state-of-the-art zero-knowledge tools are now practical in realistic settings.

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